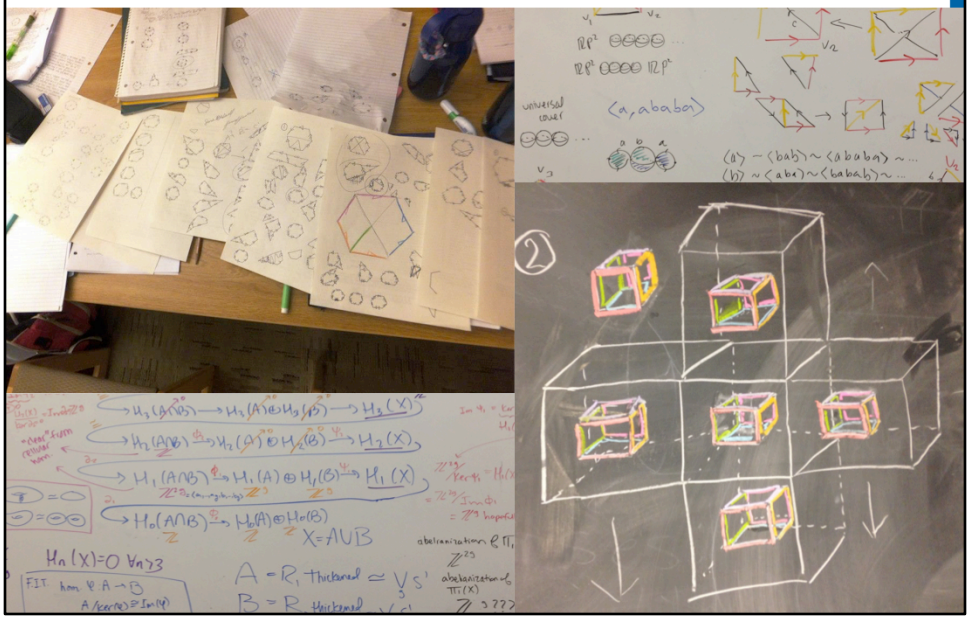
WHY DOES A DONUT EQUAL A COFFEE MUG? & OTHER QUESTIONS:

WHAT IS TOPOLOGY?

SARAH BLACKWELL

WHAT IS TOPOLOGY?



What is topology??? These are all pictures of “topology,” specifically, homework problems I’ve done. So maybe topology looks something like this. But before I can really answer the question “what is topology,” I have to ask a different question...

WHAT DOES IT MEAN FOR TWO THINGS TO BE THE SAME?

1 = 7 (mod 6)

"color" versus "colour"

congruent or similar triangles

What does it mean for two things to be the same? A big theme in math is the notion of sameness- when are two things the "same", or what does it mean for two things to be the same?

*Color v. colour- not literally the same (one has a "u" and one doesn't), but same entry in the dictionary, so we say it's the "same" word.
*1 and 7 are different numbers, but mod 6 they are the same.
*Two congruent triangles (rotation and translation) or even two similar triangles (add scaling).

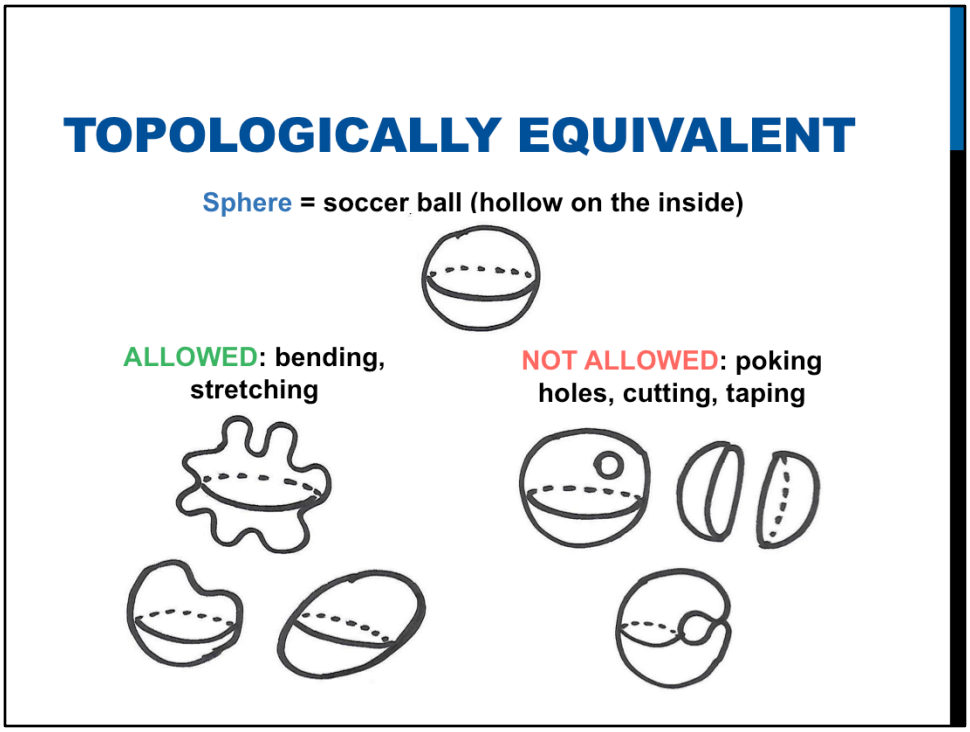
TOPOLOGY IS A WORLD WITH ITS OWN NOTION OF SAMENESS.

studying shapes and spaces

“topologically equivalent”

Topology is a world with its own notion of sameness.

Topology is a subfield of math that studies shapes and spaces (here are examples of some spaces, called a sphere, torus, and two-holed torus- we’ll talk about the sphere and torus). And, topology has its own notion of sameness; in the world of topology, we call being the same “topologically equivalent.”

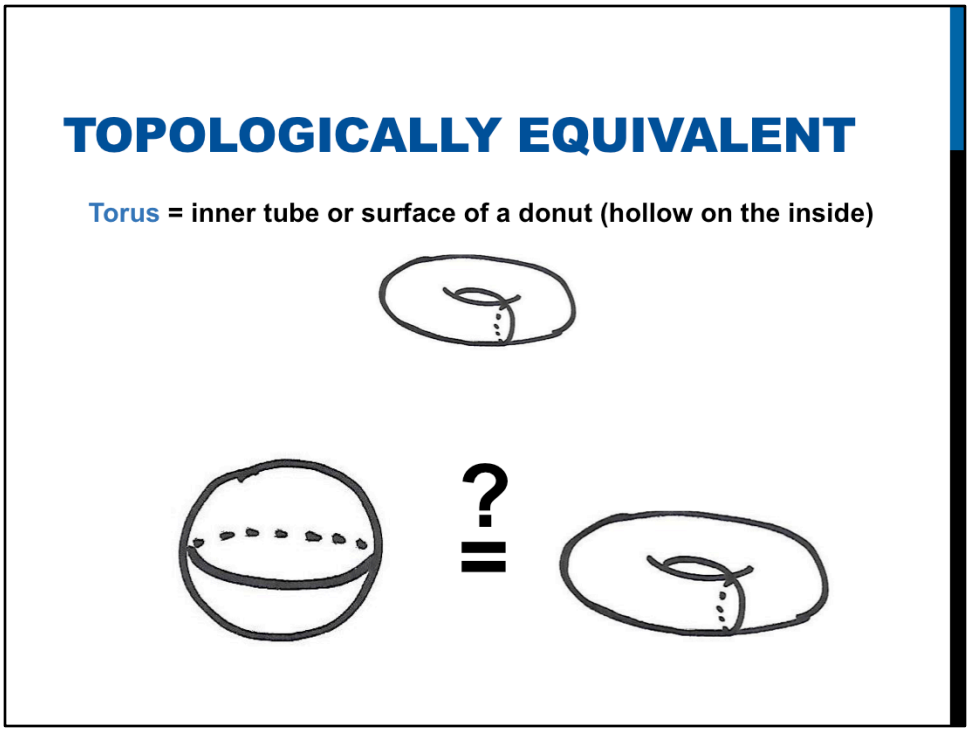


What does “topologically equivalent” mean? Let me give an example. A sphere is like a soccer ball- it is a ball that is hollow, or has air on the inside. Now imagine this sphere, or soccer ball, is made out of rubber. We are allowed to do certain things to the sphere, like bend and stretch. The shapes we come up with when we bend and stretch the sphere are called “topologically equivalent” to the sphere. There are some things we are not allowed to do, like poke holes, cut, or tape. The shapes we come up with when doing these things are not “topologically equivalent” to the sphere.

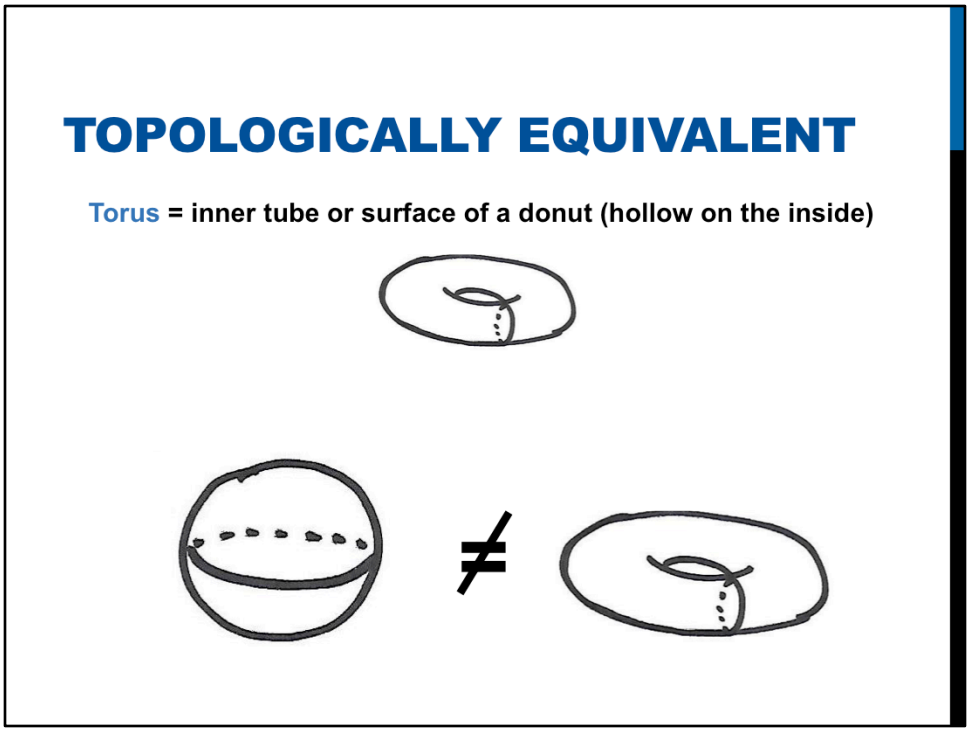
TOPOLOGICALLY EQUIVALENT

By User:LucasVB - Own work, Public Domain, https://commons.wikimedia.org/w/index.php?curid=1236079

Here is one example of two spaces that are topologically equivalent. This talk is titled "why does a donut equal a coffee mug?" Really, what I mean is these shapes are topologically equivalent (see picture and video & explain).



Now I'd like to describe another topological space- the torus. It is often described as "the surface of a donut." I think a better way to describe it is an inner tube, since like an inner tube, a torus is hollow or has air on the inside. Question- is a torus topologically equivalent to a sphere? (No- there is no way to bend or stretch the sphere so that it will have a hole in it, like the torus.)



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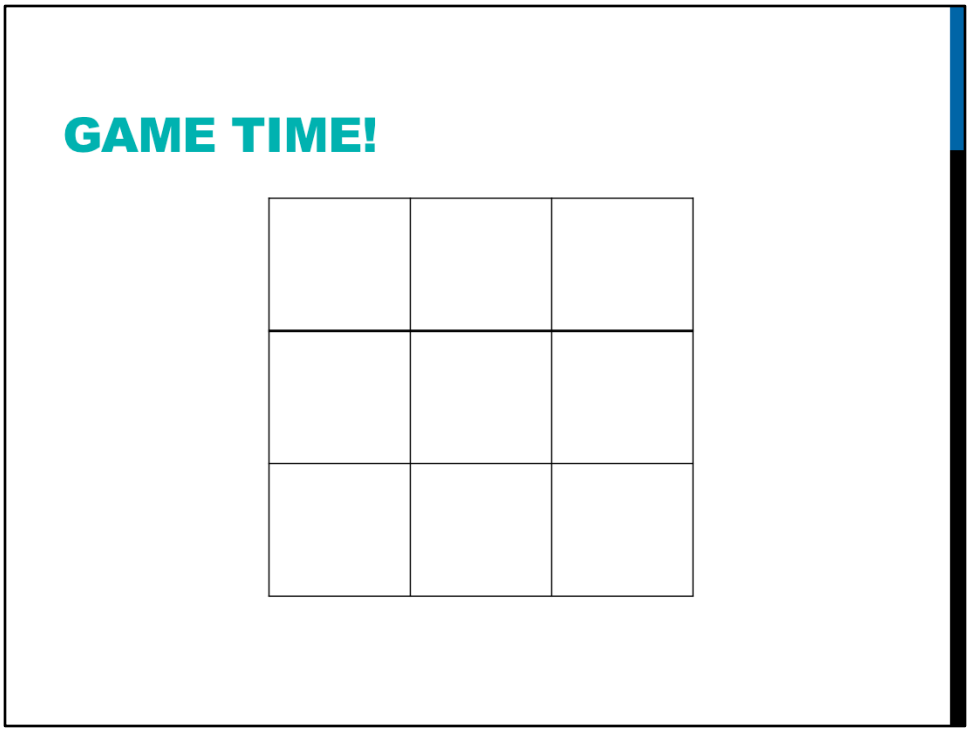
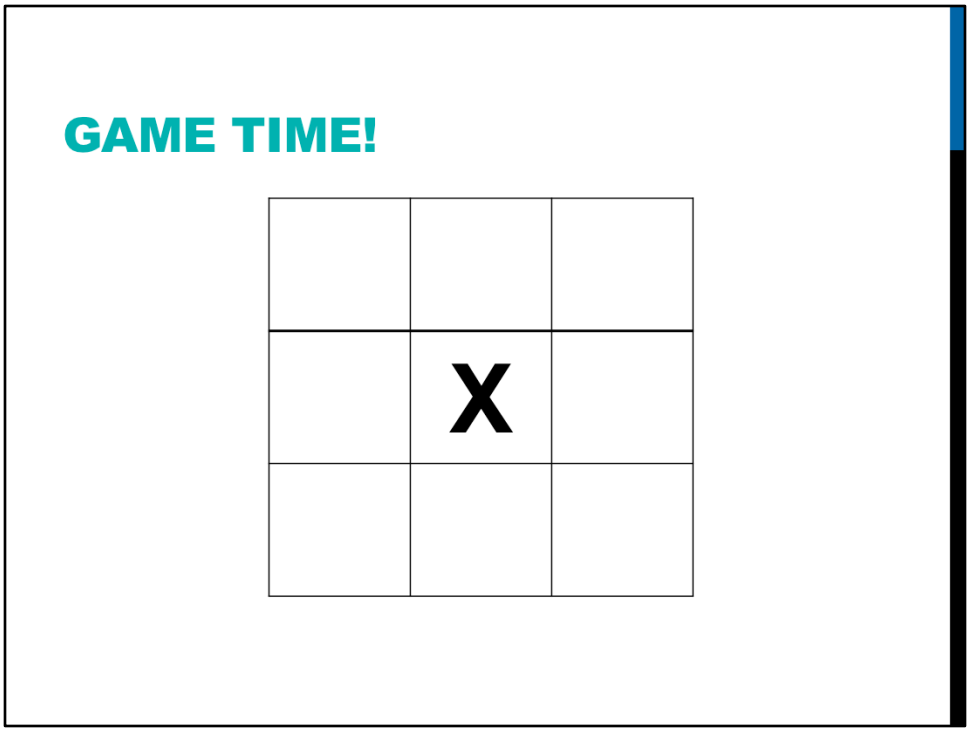


Now that I've told you a little bit about what "topology" is, we're going to play a game! Is everyone familiar with tic-tac-toe? In the usual game, two people take turns placing x's and o's in a grid, and the first person to make three in a row wins. Usually, "three in a row" can look like...



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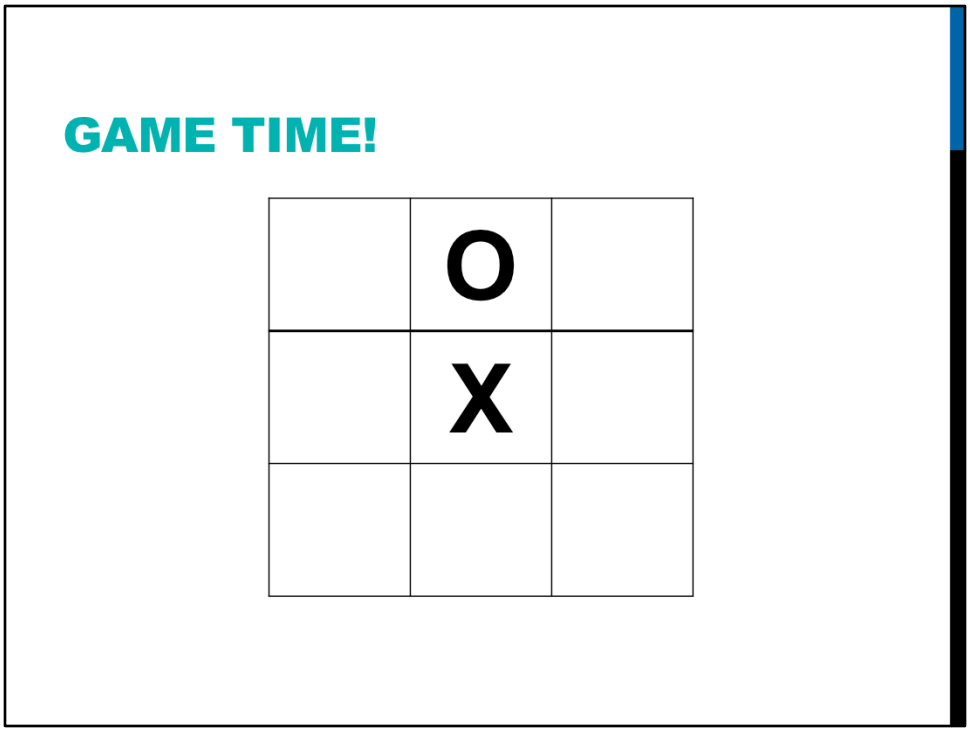

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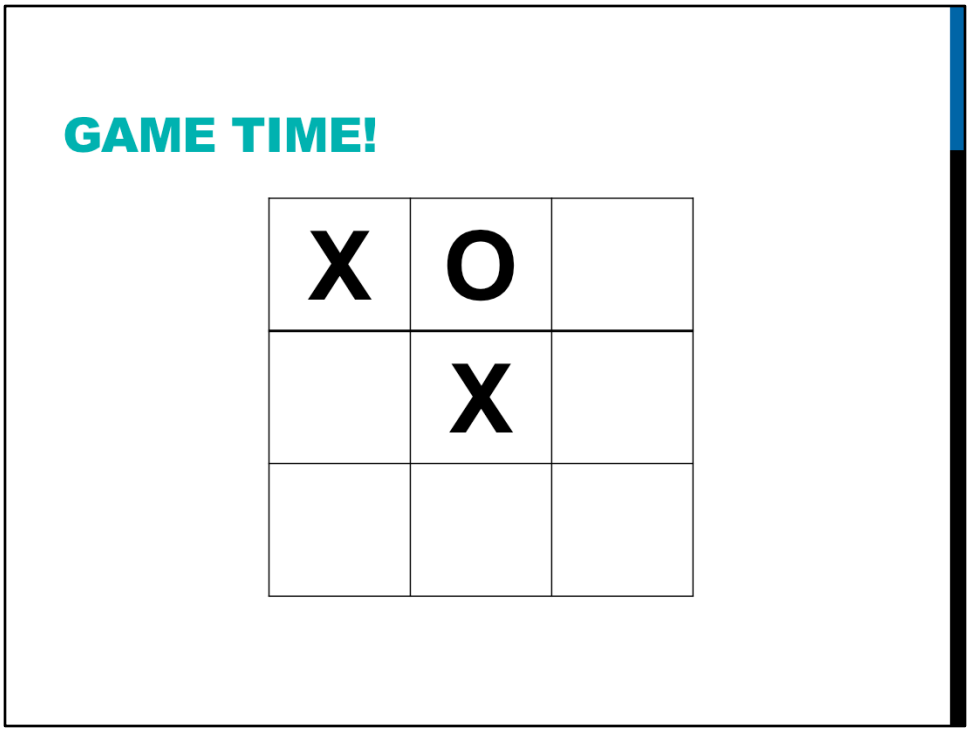

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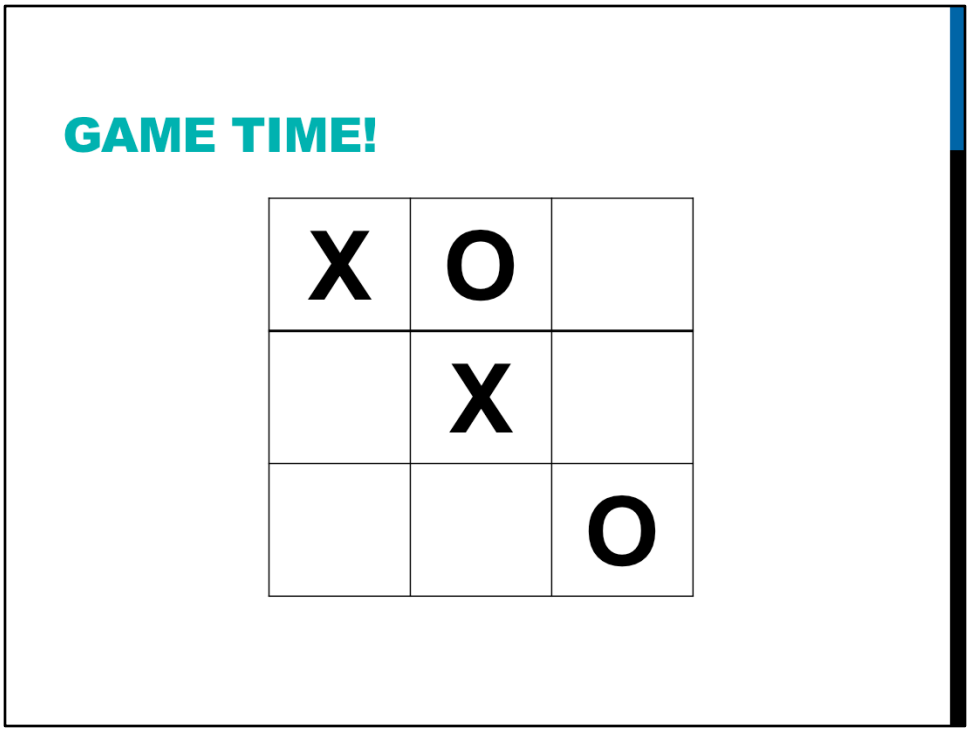

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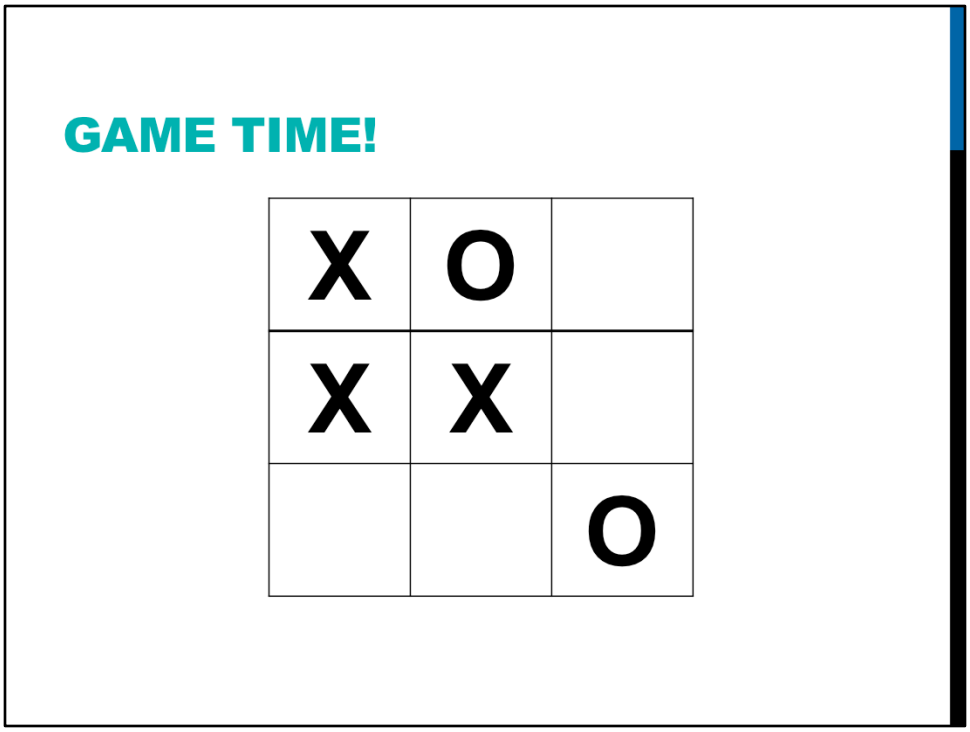

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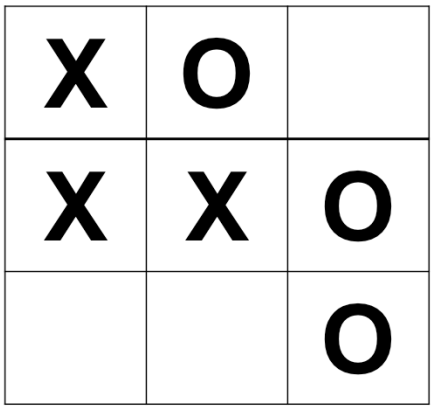
GAME TIME!



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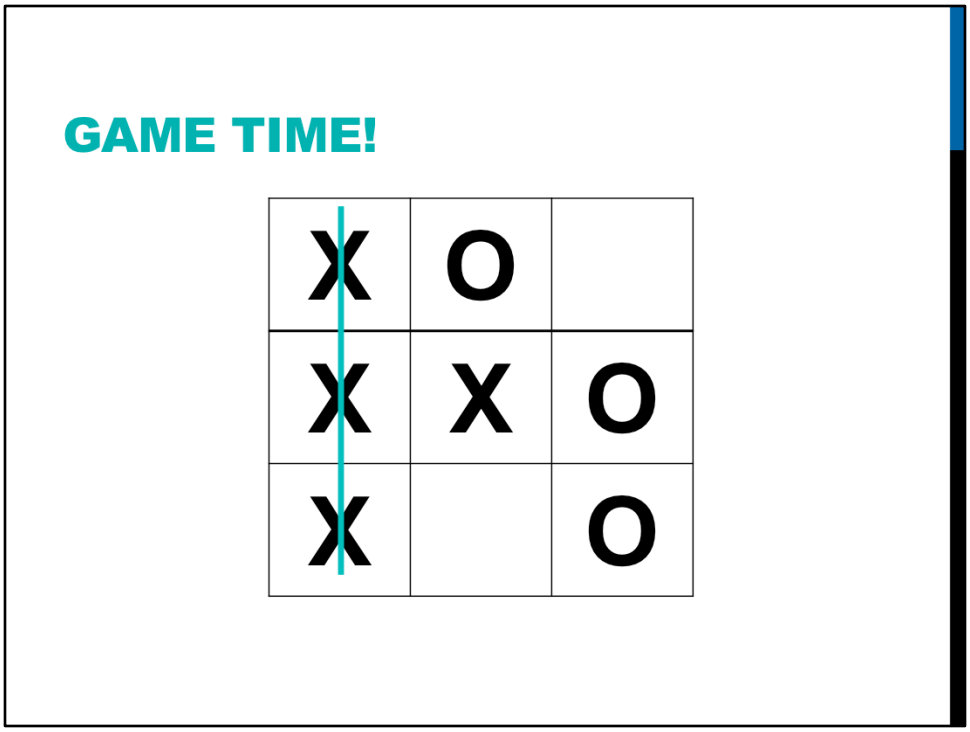

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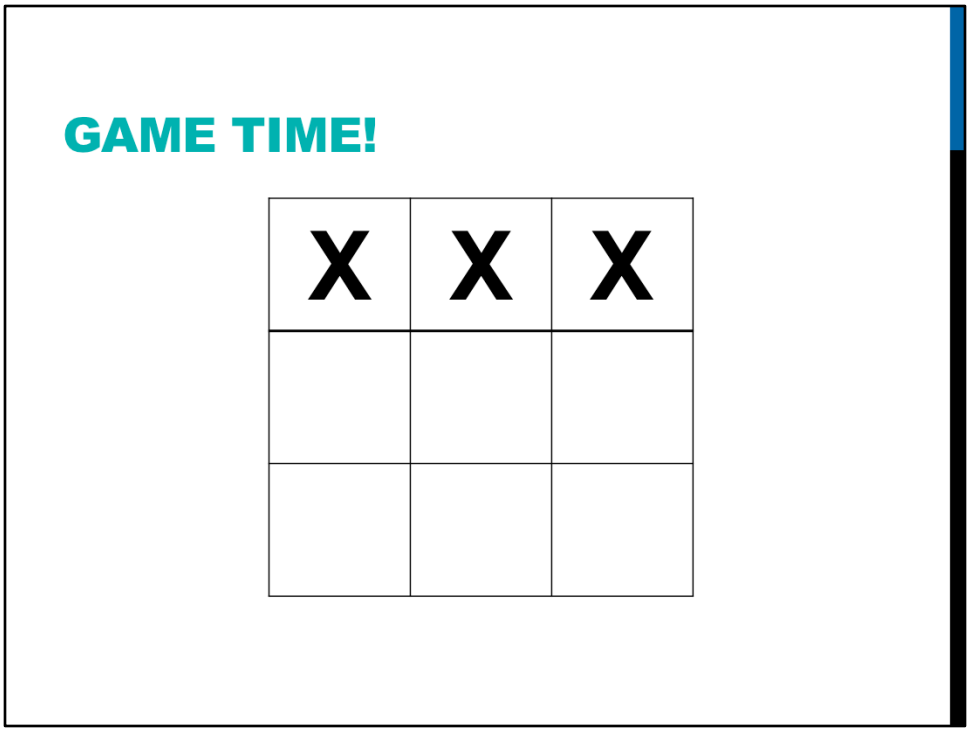
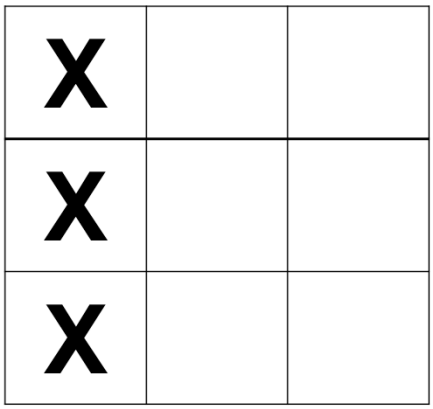
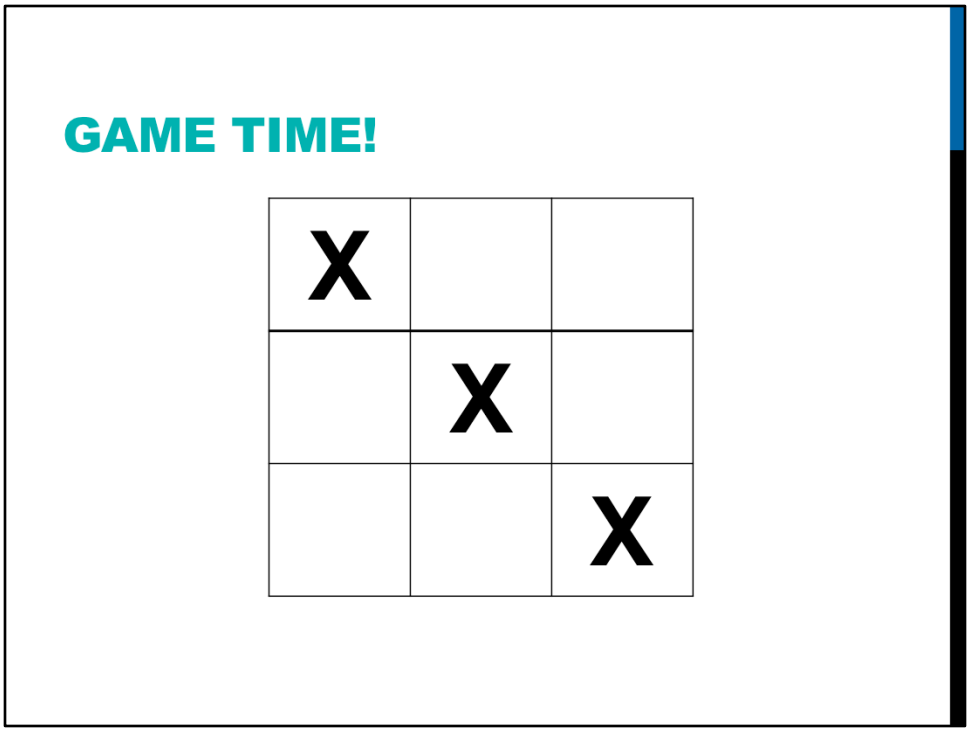


This (horizontal)...

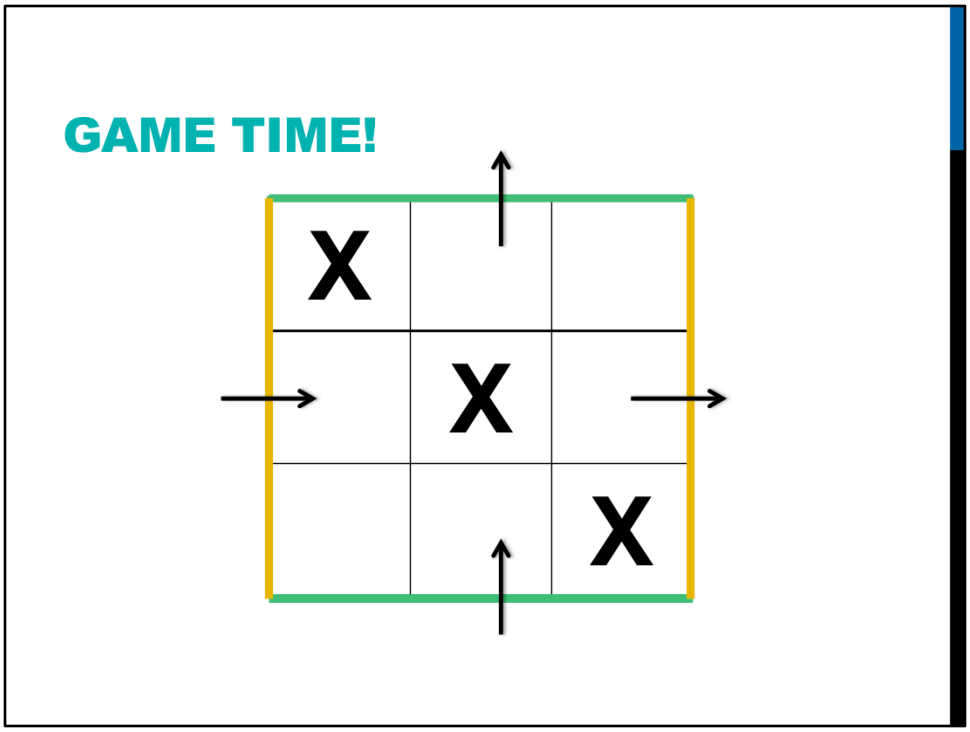
GAME TIME!



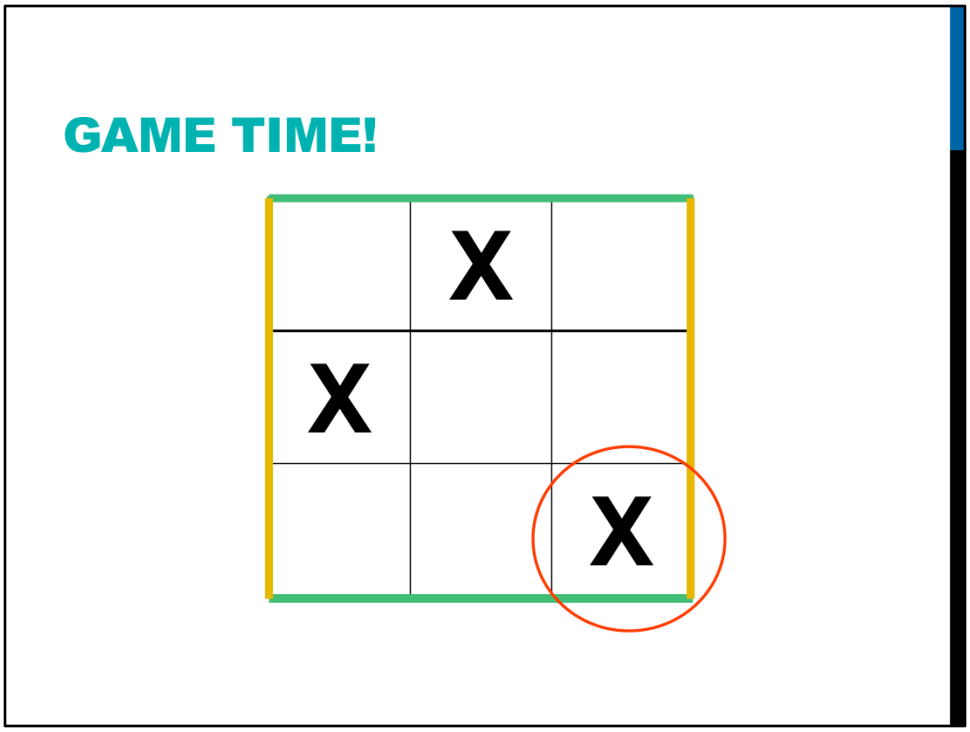
This (vertical)...



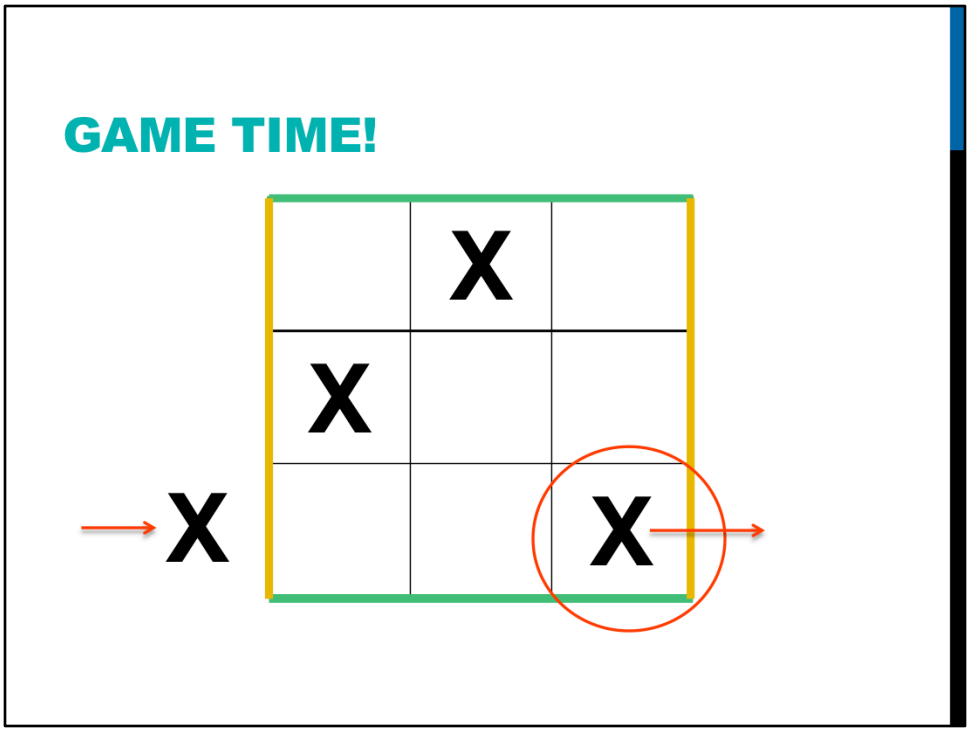
Or this (diagonal). Now I want to add a new rule.



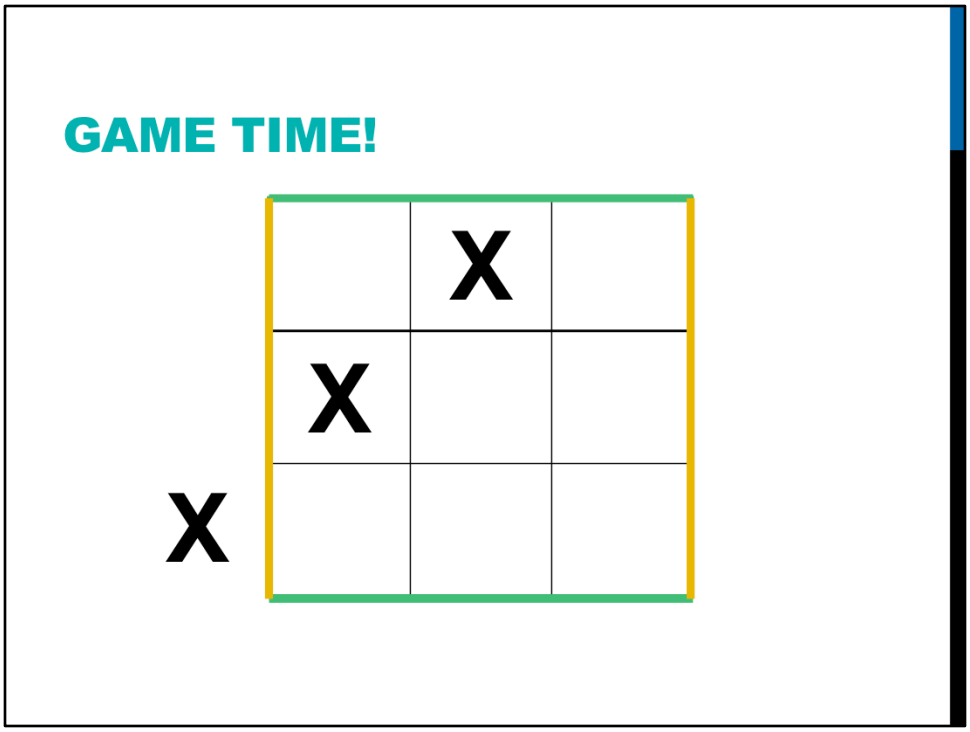
Let's pretend that we are "gluing" opposite sides of the grid together (show on diagram- the orange sides go together and the green sides go together...that is, the orange sides become the same line, and the green sides become the same line). What that means is if I go off the right side (point to grid), I come out over on the left side (point again). And if I move up (point to grid), then I come out down below (point again). This is like the arcade game "Asteroids;" when you move off one side of the screen, you appear on the other side. How does this change the game? Everything else is the same, except there is a new way you can win.



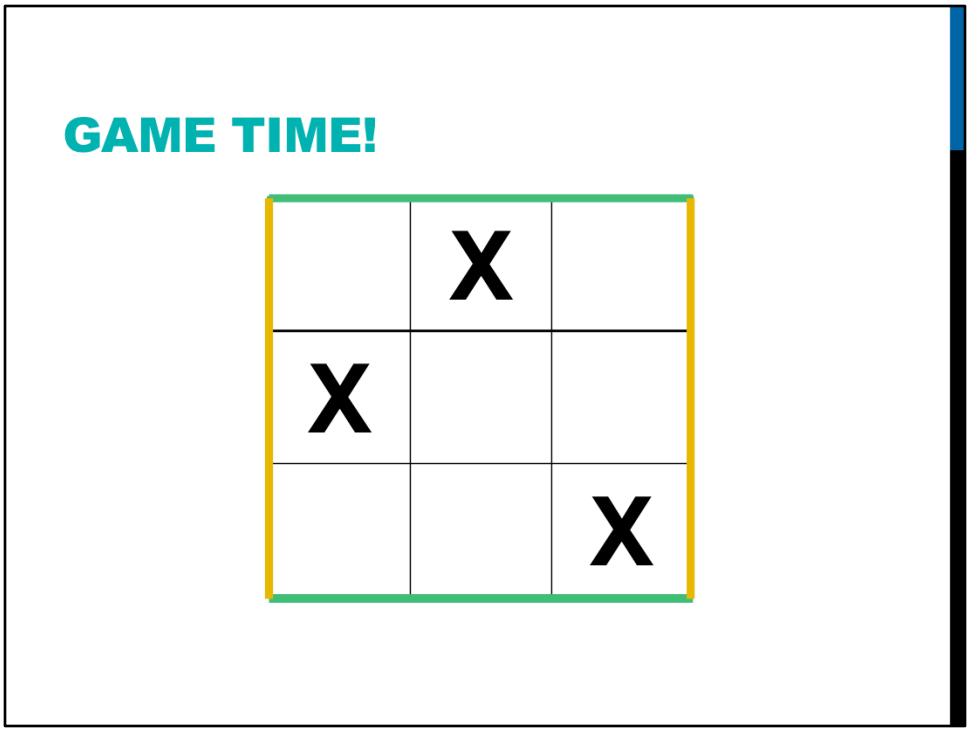
These x's now count as being "in a row." Why? Remember the orange sides are the same. So this X on the bottom...is also over here.



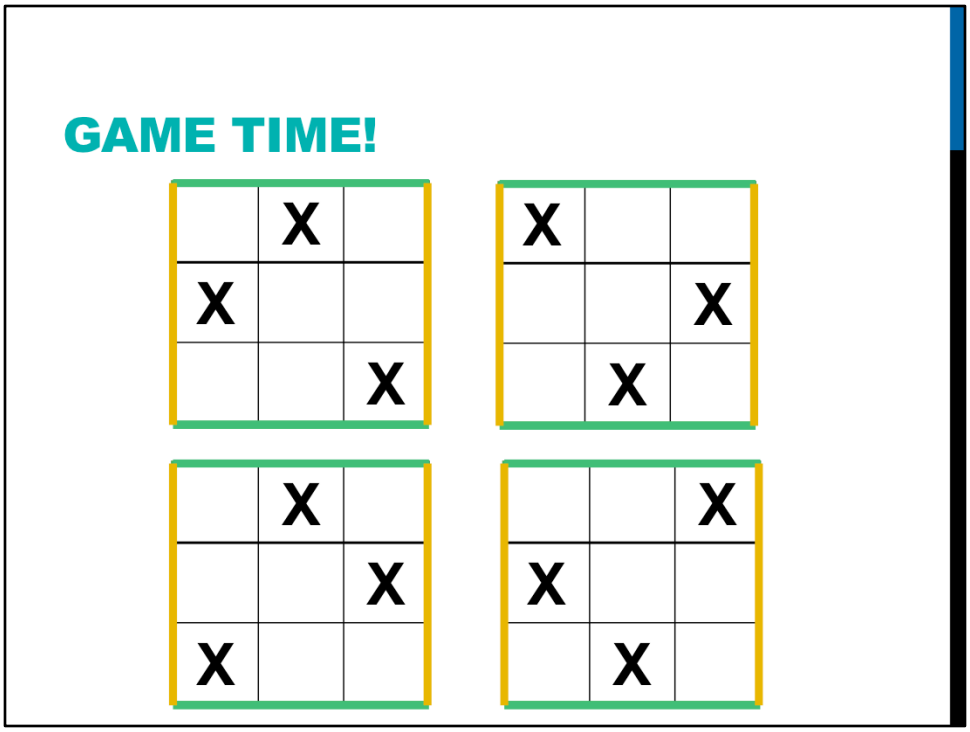
...is also over here. (Both are on the bottom and to the left of the orange line.)



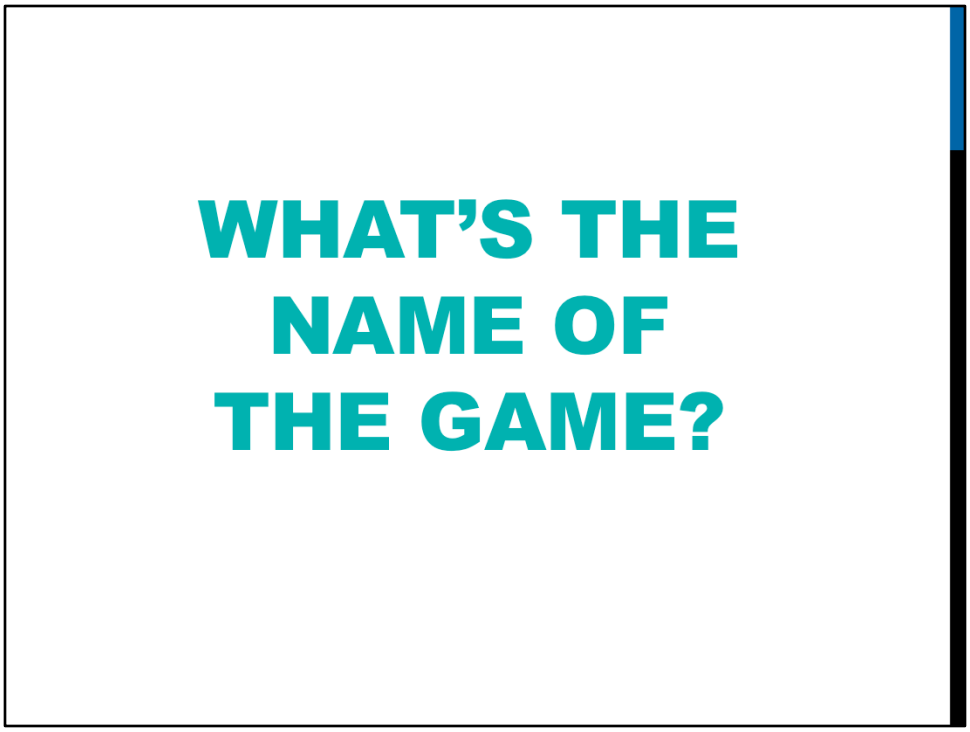
Does this make sense?



So here is one new way to win. Can you see more? I'm going to give you 5 minutes to play a few rounds with the person sitting next to you (handouts). Raise your hands if you have any questions!



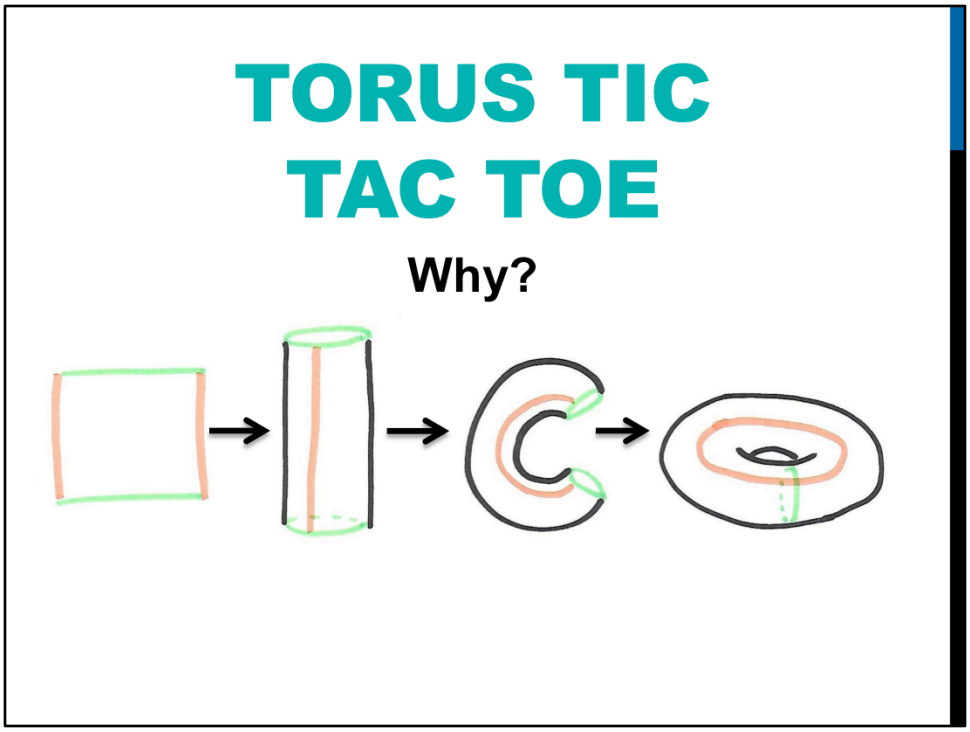
As you may have seen while playing, here are all of the new ways to win with this new rule. I claim that there are no other ways to win, but I'll let you think about why that is true! Another interesting fact- unlike traditional tic-tac-toe, this game will never end in a draw. That is, one of the players will always win! Why? Take home question to think about...



I never told you the name of the game! What is it?

Torus tic-tac-toe! Does anyone know why? What does a torus have to do with anything?

If you have a stretchy rubber square, and you actually glue the sides together (like we pretended), you get a torus! So, torus tic-tac-toe is tic-tac-toe on a torus!



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PICTURE REFERENCES

- **Slide #4**
  - https://ncatlab.org/nlab/show/Introduction+to+Topology
- **Slide #6**
  - https://www.cems.riken.jp/en/laboratory/qmtrt
- **Slide #30**
  - https://www.amazon.com/Shape-Space-Chapman-Applied-Mathematics/dp/0824707095

That's all I have! Here are my picture references...

Torus tic-tac-toe came from the book *The Shape of Space*, which was my first topology book. It's a very approachable and fun book, and a great place to start if you want to learn more!