Researcher Bios

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Professional Title: PhD student

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Lesson Plan Title, Grade Level, and Keywords:
Title: Torus Tic-Tac-Toe
Grade level: 6th-8th
Keywords: mathematics, geometry, symmetry, reasoning

Brief Description of Research Interests: My research is in topology, a field of mathematics. Topology is similar to geometry, in that it involves studying shapes and spaces, but in topology the notion for what it means for two spaces to be the "same" is a bit looser. For instance, topologists don't think about lengths and angles as a geometer might. I am primarily interested in four-dimensional spaces and lesser-dimensional spaces sitting inside these spaces. More specifically, I am interested in trisections of 4-manifolds, group trisections, Lagrangian cobordisms between Legendrian knots, and ways to make connections between these things.

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Lesson Plan Title, Grade Level, and Keywords:
Title: Torus Tic-Tac-Toe
Grade level: 6th-8th
Keywords: mathematics, geometry, symmetry, reasoning

Brief Description of Research Interests: I like to think about topology and geometry using tools from algebra. I really like to study shapes in multiple dimensions. I find it fascinating how there is quite a lot of structure in shapes, if you think about them a bit abstractly. I feel that knowing the structure of shapes (i.e. the "rules" they follow) is important as it has applications in many fields like data analysis in computer science, protein folding in biology and theoretical physics.

Lesson Plan Information Sheet

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Introduction/Abstract to Lesson Plan (max. 100 Words) Include aspects of the lesson that are unique and innovative.	This lesson plan gives students a glimpse into what mathematics researchers do in a fun and engaging manner. To make the experience novel a 3D shape, the torus, is introduced by redesigning a familiar game, Tic-Tac-Toe. This upgraded version of Tic-Tac-Toe gets students to think about the torus and its symmetrical properties. It also introduces students to the idea of gluing 2D shapes to make 3D shapes, and visualizing the 3D shape simply by observing the 2D shape. The game "Torus Tic-Tac-Toe" comes from the book *The Shape of Space* by Jeffery Weeks. (Note: while the torus is technically a 2D shape, we think of it as sitting in 3D space, as opposed to the flat square. This is why we call the torus "3D" for the purposes of this lesson plan.)
List of Standards Addressed (This should be a list of all full standards addressed by the lesson.)	**MGSE8.G.1** Verify experimentally the congruence properties of rotations, reflections, and translations: lines are taken to lines and line segments to line segments of the same length; angles are taken to angles of the same measure; parallel lines are taken to parallel lines. **MGSE8.G.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. **MGSE8.G.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Note: although our lesson plan does not correspond exactly with the *mathematical content standards* listed above, it does align with many of the *mathematical practice standards*, so we hope that teachers may still find this lesson plan useful.) **1 Make sense of problems and persevere in solving them.** This lesson plan introduces a problem which is slightly outside the content covered in middle school, yet is approachable due to its structure. Therefore, students need to think about how to make sense of this novel concept in order to engage with the questions posed. The questions are intentionally thought-provoking in nature, in order to mimic the abstract and undefined nature of mathematical research. **2 Reason abstractly and quantitatively.** Students are posed with a question that asks them to not only find winning moves in a game, but also to verify that the winning moves they

	find are the only ones possible. This requires the students to provide an argument to convince each other, which is nudging them towards proofs. **4 Model with mathematics.** The lesson plan asks students to model the situation in two ways: by drawing on a 2D square and by creating a 3D model of a torus. Students will need to understand both of these representations in order to have a full understanding of the game.
Learning Objectives using Measurable Verbs (what students will be able to do)	Students will be able to describe some of the components of mathematics research (in analog to the scientific method). Students will be able to explain how gluing the sides of a square together creates a torus. Students will be able to use a flat square diagram to represent playing tic-tac-toe on a torus, and identify new winning moves in this setting. Students will practice explaining mathematical observations verbally to a classmate and through writing.
Appropriate Grade Levels	6-8 (may also be suitable for 9-12)
Group Size/# of students activities are designed for	2
Setting (e.g. indoors, outdoors, lab, etc.)	Indoors (Note: we envisioned this lesson plan taking place in an in-person classroom, but we think it could be modified to work in an online setting as well. Each student would receive a torus kit and make their own torus, rather than in pairs. Also, students would need some way to practice playing the game together on Zoom, such as a shared virtual whiteboard.)
Approximate Time of Lesson (Break down into 20-50 minute periods)	Total: 50-75 min Introduction: 5 min Background: 5-10 min Activity: 30-40 min Reflection: 10-20 min (Note: the time intervals can be adjusted to fit two 50 minute classes or one 75 minute class.)
Resources Needed for Students (e.g. scissors, paper, pencils, glue, etc.)	• Torus kit: felt rectangle with numbered grid drawn on, one long Velcro strip, two short Velcro strips, ten Velcro circles, five felt X's and O's (small squares with letters drawn on) • Pencils and erasers • Colored pencils, markers, or highlighters could be useful to students for part 2 of the activity • Scratch paper

<table>
<tr><td>Resources Needed for Educators (e.g. blackboard, Powerpoint capabilities, etc.)</td><td>Blackboard or Powerpoint (for brief lesson)
Printer/copier (for handouts)</td></tr>
<tr><td>Apps/Websites Needed</td><td>Our lesson plan does not make use of this website, but as an FYI, the author of the book we cited in our abstract also made a website where you can download various torus games, including Torus Tic-Tac-Toe: http://www.geometrygames.org/TorusGames/.</td></tr>
<tr><td rowspan="4">Lesson Activity (step by step description of activity)</td><td>Introduction (5 min)
Brief introduction (slides- includes notes and animations)
What does mathematics research look like?</td></tr>
<tr><td>Background (5-10 min)
Brief lesson (slides- includes notes and animations)
What is a torus?
Rules of Torus Tic-Tac-Toe</td></tr>
<tr><td>Step by Step Activity (30-40 min)
Part 1: building the torus (front of handout)
Group students into pairs
Each pair gets a torus kit (prepared ahead of time)
Students will follow the instructions on where to place the Velcro, and then join up the Velcro strips to make a torus
We suggest having extra materials (such as the Velcro strips) on hand in case students make a mistake and need more
Part 2: practicing the game (back of handout)
Ask students to play a few rounds of torus tic-tac-toe with their partner
Encourage students to use the felt torus they built previously as a visual aid to compare with the grids on the handout
Provide students with the reflection questions while they practice the game so they can start thinking as they see how the game works</td></tr>
<tr><td>Reflection/Assessment (10-20 min)
Part 3: reflection questions (back of handout)
Ask students to reflect on the following questions in the same pairs as before. Then come together for a class discussion and conclusion.
Explain how gluing the sides of a square together makes a torus.
Describe the types of symmetries you see in the square. Describe the types of symmetries you see in the torus. How do they compare and contrast?
What are all the new ways you can win in Torus Tic-Tac-Toe?
Unlike traditional Tic-Tac-Toe, this game will never end in a draw (that is, one player will always win). Why?
(Challenge) What other games could you modify in a similar way (by gluing the sides of the board together)? How would the new rule change these games?
(Note: question #2 could be modified to relate more directly to a past lesson on symmetry.)</td></tr>
</table>

Final Product/Assessment (e.g., quiz, presentation, essay, etc.)	Ask students to pick two or three of the discussion questions to write about in a (homework) reflection. We recommend that students hand-write their reflection so that they can include pictures. (And students should be encouraged to include pictures!) We anticipate that these questions might be very difficult for students, as some are more open-ended, so be sure to let students know that it is okay if they don't write a complete answer. The goal is for students to think and write about relatively open-ended questions, just like a mathematics researcher would.