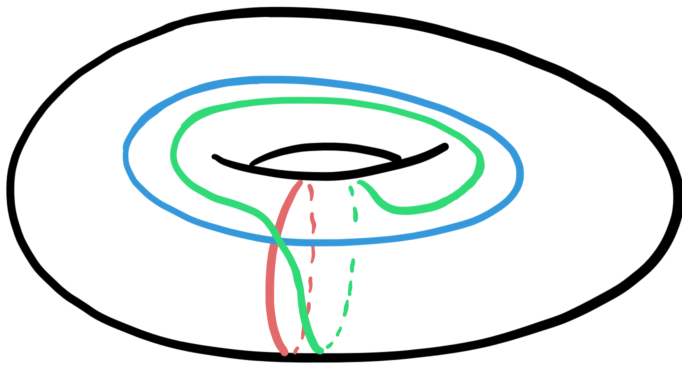
TRIPLE KNOT GRID DIAGRAMS

Sarah Blackwell
University of Georgia

SETUP: (1,0)-TRISECTION OF $\mathbb{CP}^2$

decomposition into three 4-balls
("0")



triple intersection of 4-balls
("1")

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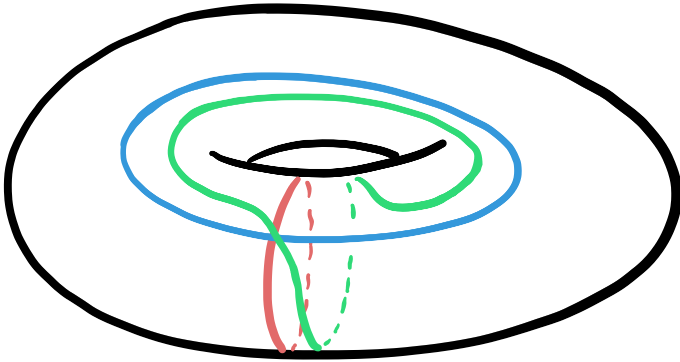

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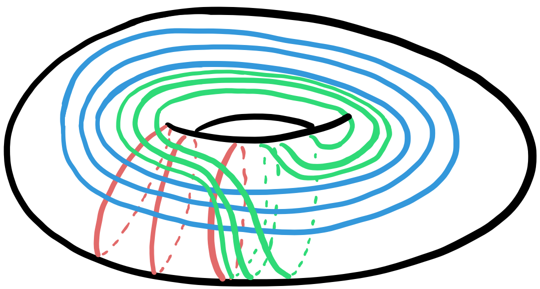

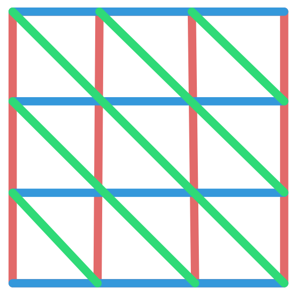

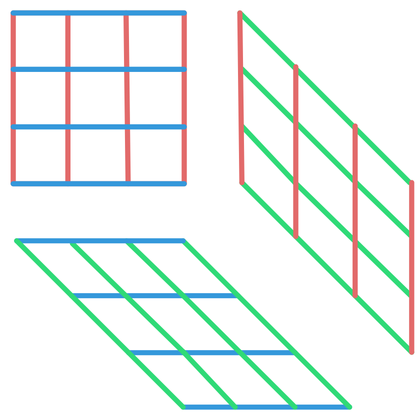

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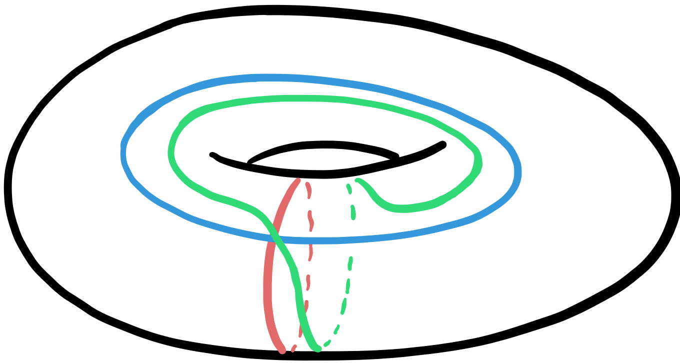

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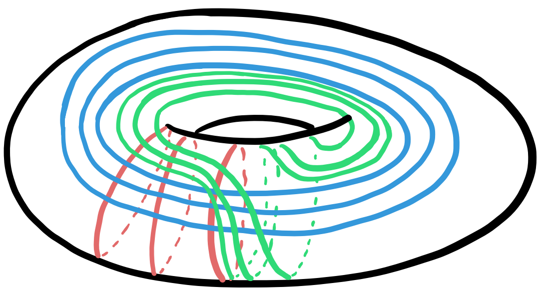
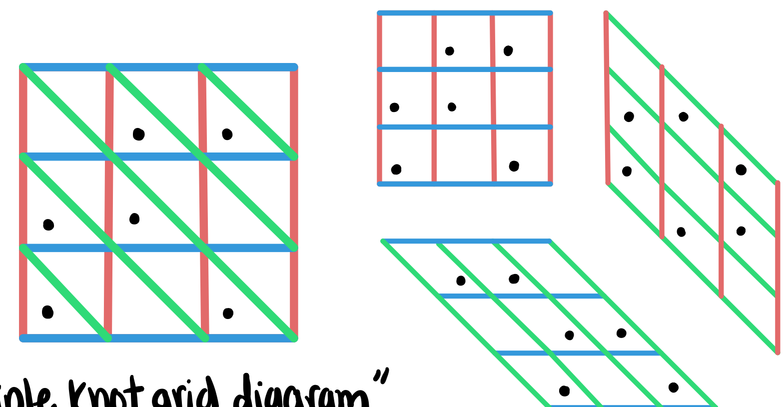
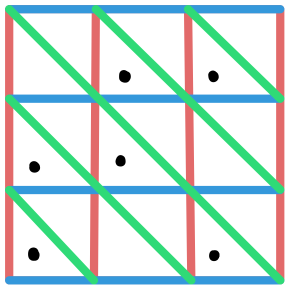




"triple knot grid diagram"

0 or 2 dots in every row, column, and diagonal

SURFACES IN $\mathbb{CP}^2$



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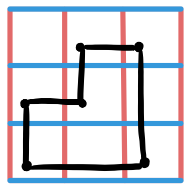
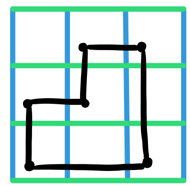
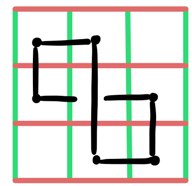


red over blue



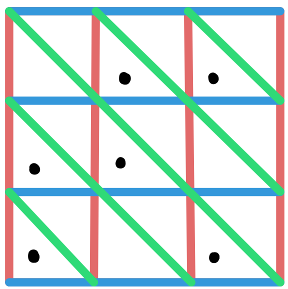
blue over green



green over red

cap off links w/ disks $\rightsquigarrow$ surface in $\mathbb{CP}^2$

SURFACES IN $\mathbb{CP}^2$



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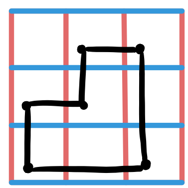

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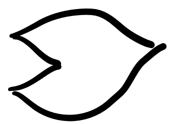

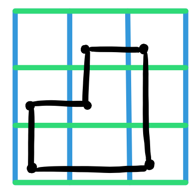

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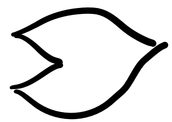

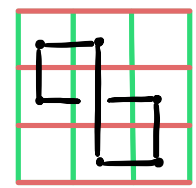

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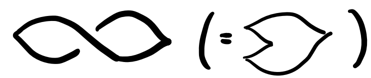

turn 45° counterclockwise ⇝ Legendrian front

~~cap off~~ fill Legendrian links w/ disks ⇝ "Lagrangian-like" surface in $\mathbb{CP}^2$??

SURFACES IN $\mathbb{CP}^2$

conjecture: for every triple grid diagram where each component is a max tb Legendrian unknot, there is a Lagrangian in $\mathbb{CP}^2$ which intersects the pieces of the (1,0)-trisection as specified by the diagram

conjecture: every Lagrangian in $\mathbb{CP}^2$ is Hamiltonian? Lagrangian? isotopic to one given by a triple grid diagram

conjecture: every connected, oriented triple grid diagram in which each grid is a Legendrian unlink of max tb Legendrians gives a torus

SURFACES IN $\mathbb{CP}^2$

conjecture for every triple grid diagram where each component is a max tb legendrian unknot, there is a Lagrangian in $\mathbb{CP}^2$ which intersects the pieces of the (1,0)-trisection as specified by the diagram

conjecture every Lagrangian in $\mathbb{CP}^2$ is Hamiltonian? Lagrangian? isotopic to one given by a triple grid diagram

conjecture every connected, oriented triple grid diagram in which each grid is a Legendrian unlink of max tb Legendrians gives a torus

Evidence...

- have examples of the standard embedded $\mathbb{RP}^2$ (three max tb unknots) and an immersed S^2 with one double point (two max tb unknots and a Hopf link of max tb unknots)
- have no embedded oriented surfaces with genus $\neq 1$ (with max tb unknots)

MINIMAL TRIPLE GRID

$n \times n$

- of a smooth link L:
  smallest n st $\exists$ an $n \times n$ triple grid diagram with L as one of its links
- of a Legendrian link L:
  smallest n st $\exists$ an $n \times n$ triple grid diagram with L as one of its links
- of a triple of Legendrian links L_1, L_2, L_3:
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KNOWN RESULTS

the minimal triple grid number of… the smooth unknot is 2
the smooth two component unlink is 4
the smooth trefoil is 5
the smooth Hopf link is 6
the smooth three component unlink is 6
the smooth anything else is ≥ 6

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KNOWN RESULTS

the minimal triple grid number of... the Legendrian max tb unknot is 2
a triple of Legendrian max tb unknots is 2
the Hopf link with max tb unknot components is 6

THANKS!