DEMYSTIFYING THE 4th DIMENSION

Sarah Blackwell
University of Virginia

WHY DIMENSION 4?

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TOPOLOGY...

dim < 4 : "small enough"

dim > 4 : "big enough"

big open Q's: e.g. smooth 4D Poincaré conjecture

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PHYSICS...

space-time (3+1)

symplectic geometry + mechanics

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me!

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MANIFOLDS

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↑

spaces which look locally like Euclidean space

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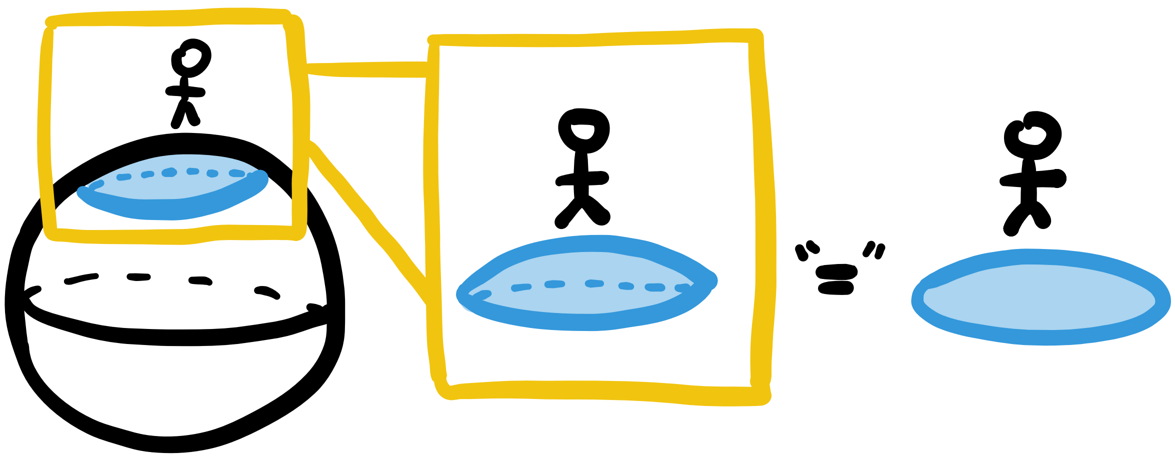

MANIFOLDS

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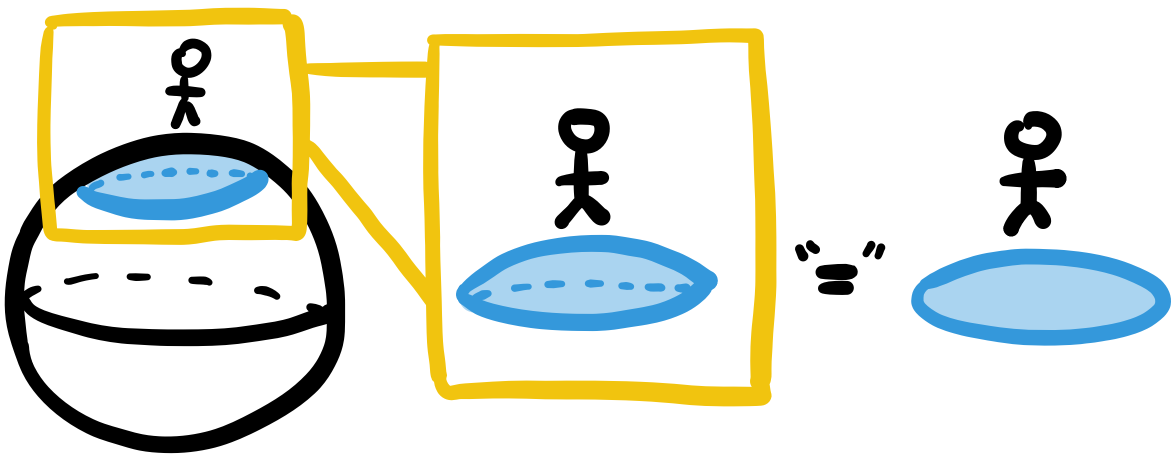

let's see some examples...

MANIFOLDS

DIMENSION 0:

MANIFOLDS

DIMENSION 0: 

1 pt

MANIFOLDS

DIMENSION 0:

1 pt

2pts

MANIFOLDS

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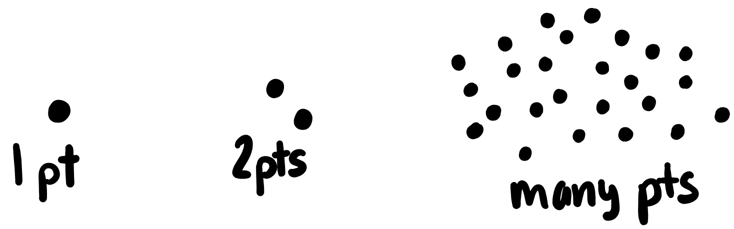

(etc)

MANIFOLDS

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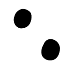
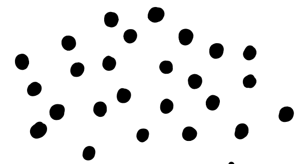


2pts



many pts

(etc)

DIMENSION 1:

MANIFOLDS

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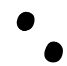
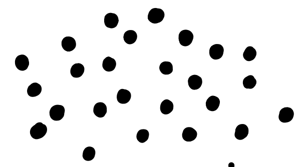


2pts



many pts

(etc)

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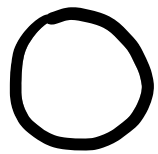

circle S^1

MANIFOLDS

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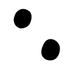

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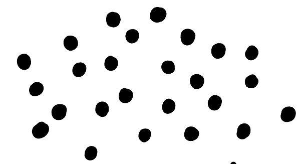

many pts

(etc)

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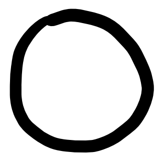

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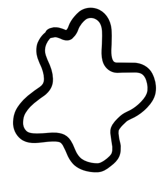

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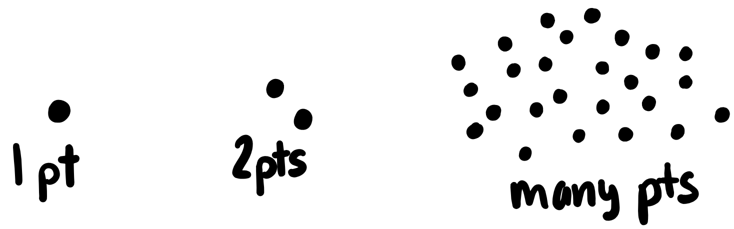

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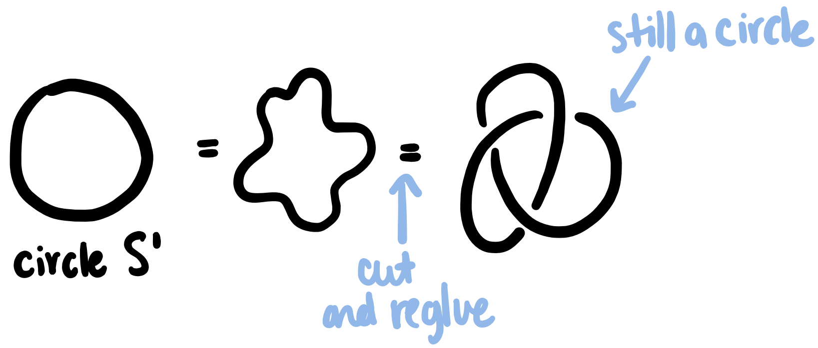

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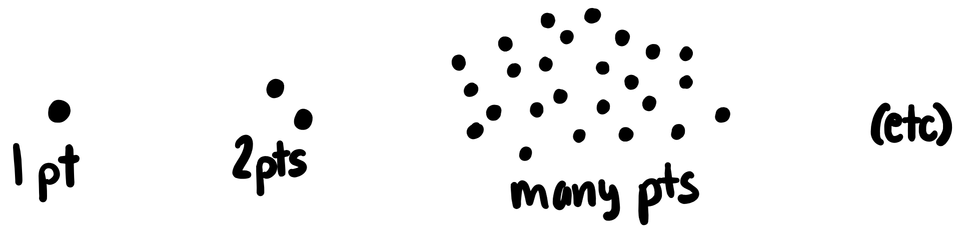

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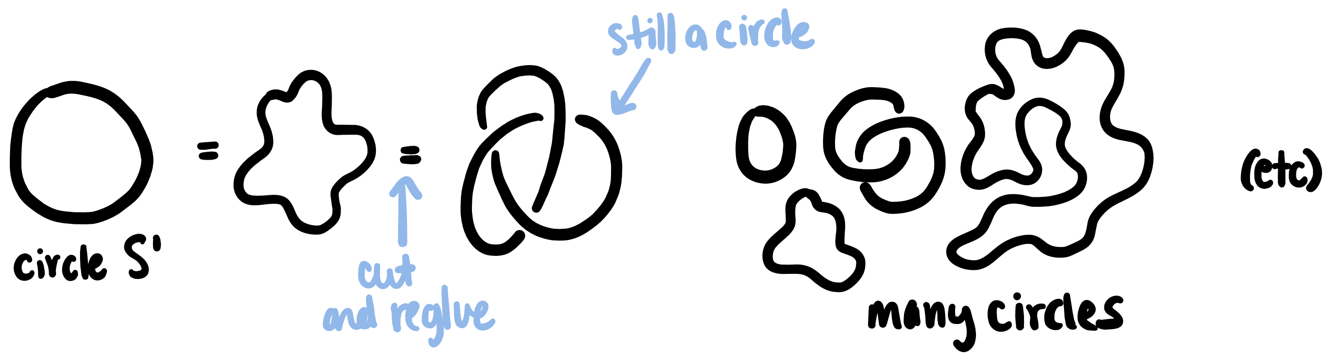

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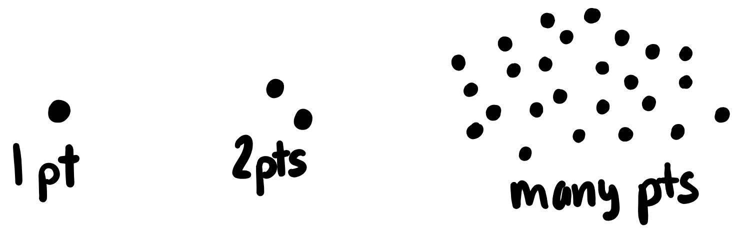

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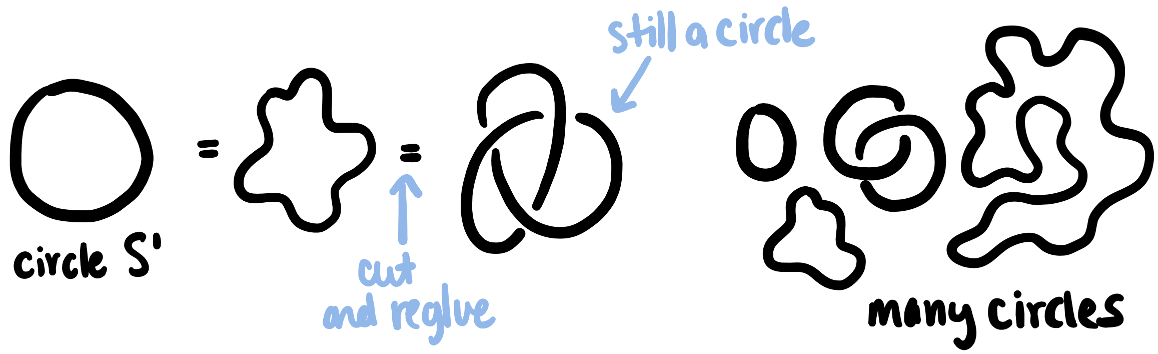

(etc)

DIMENSION 2: which are manifolds?

MANIFOLDS

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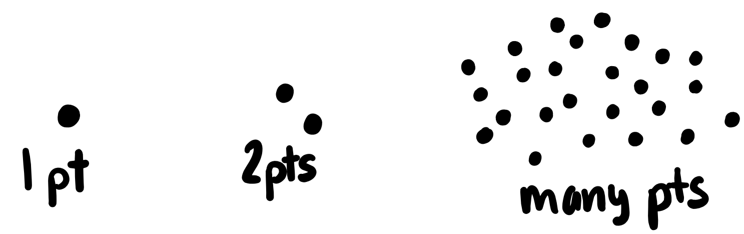

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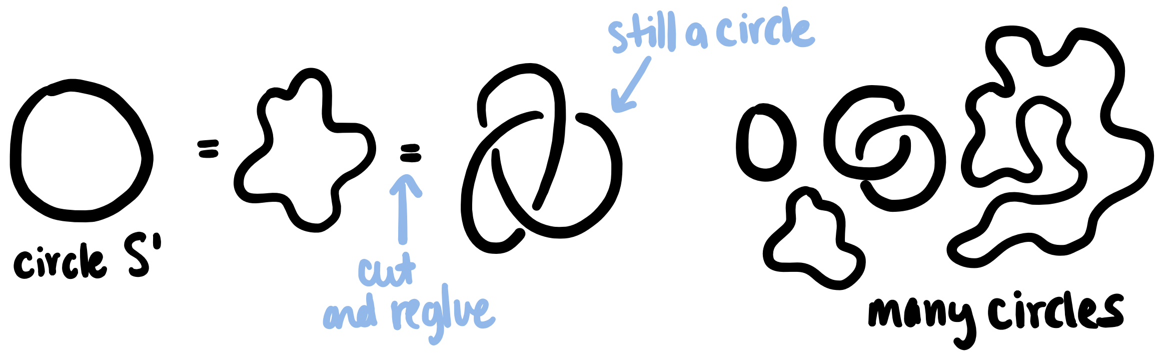

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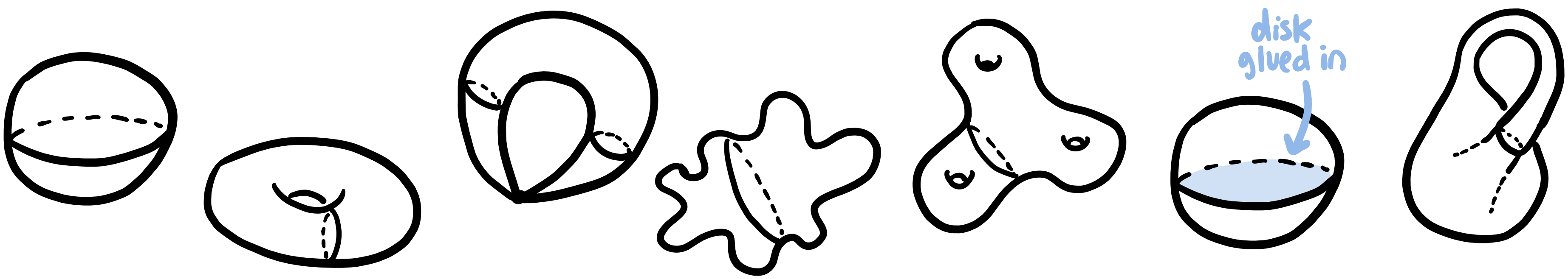

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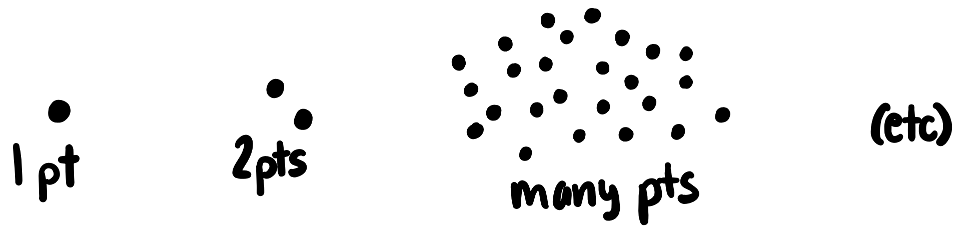

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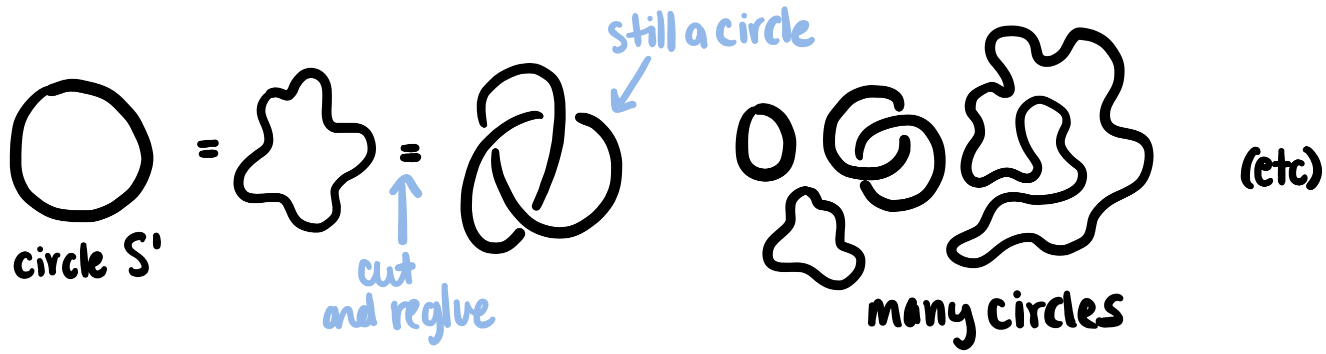

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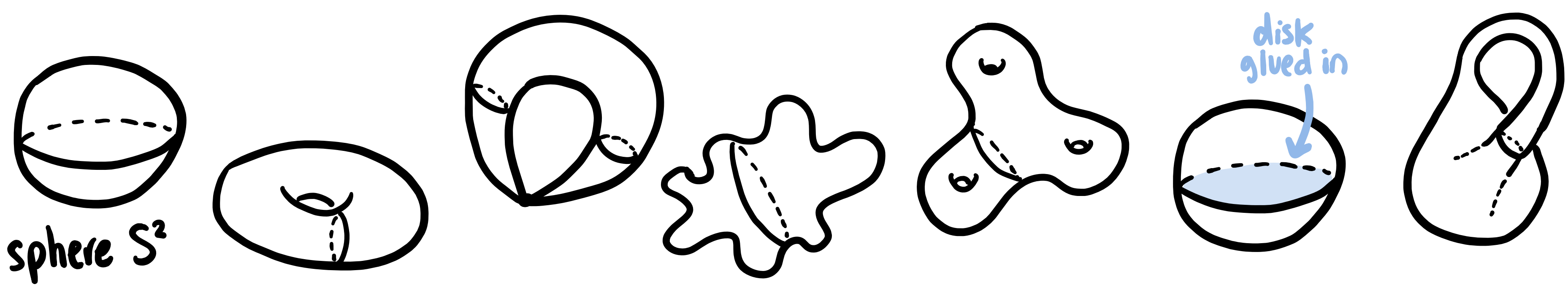

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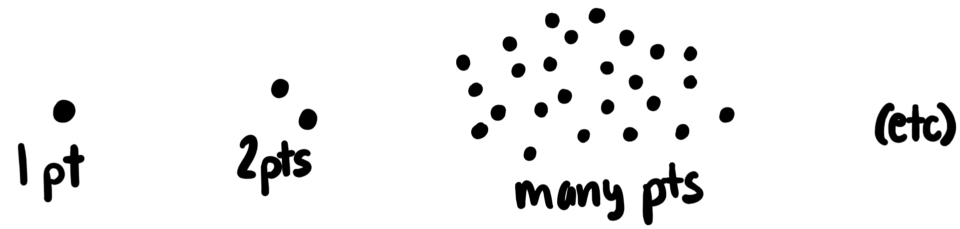

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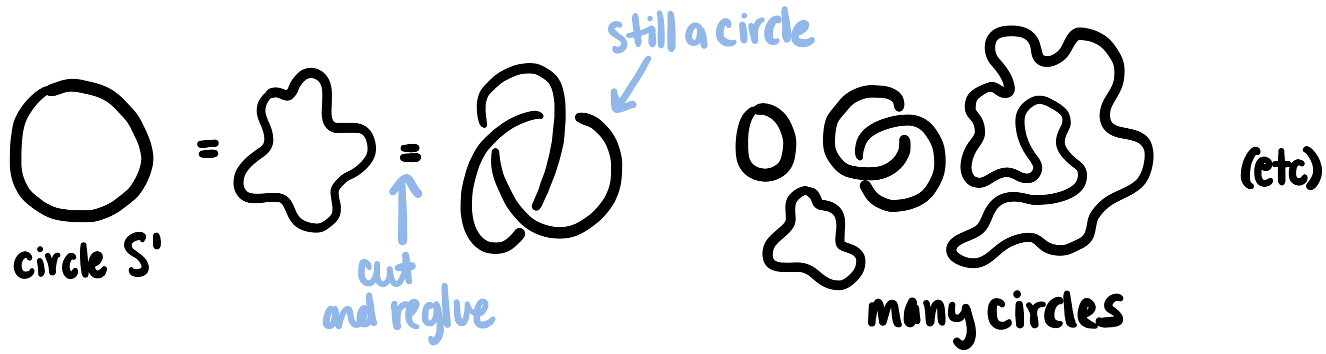

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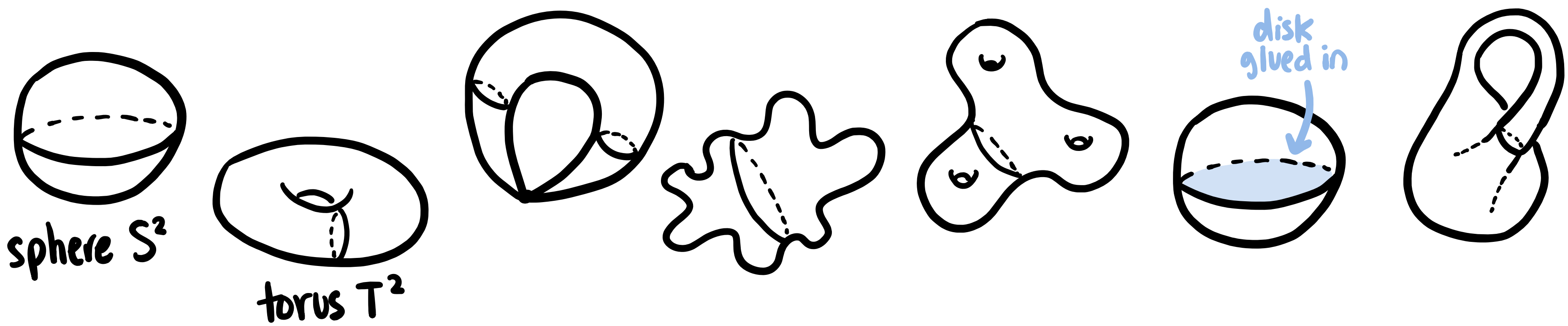

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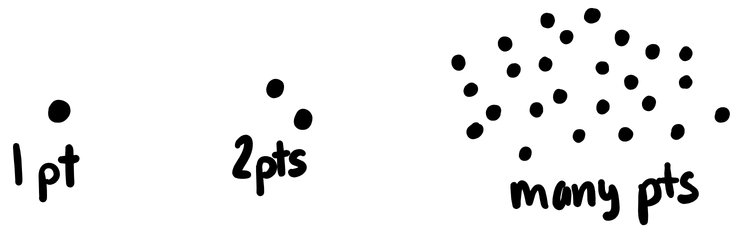

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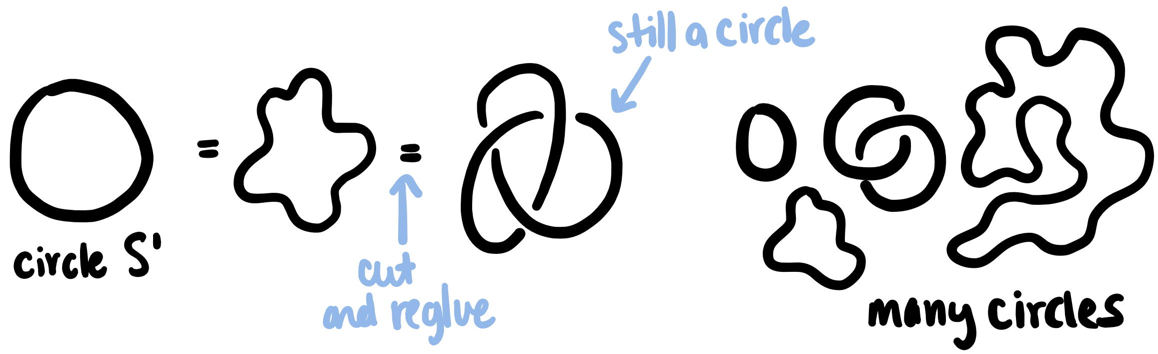

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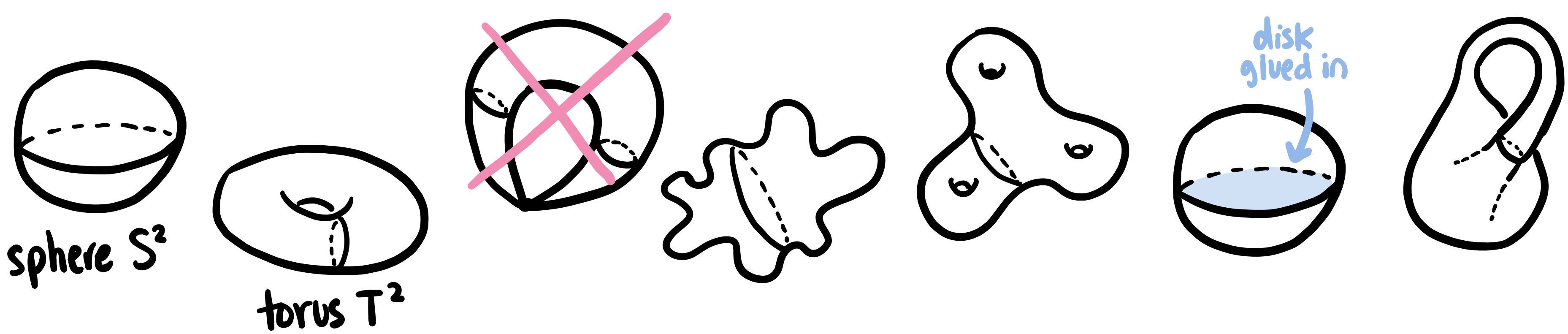

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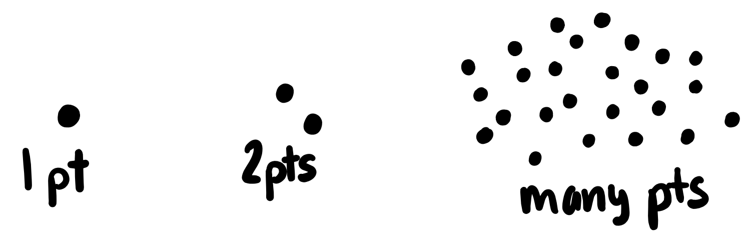

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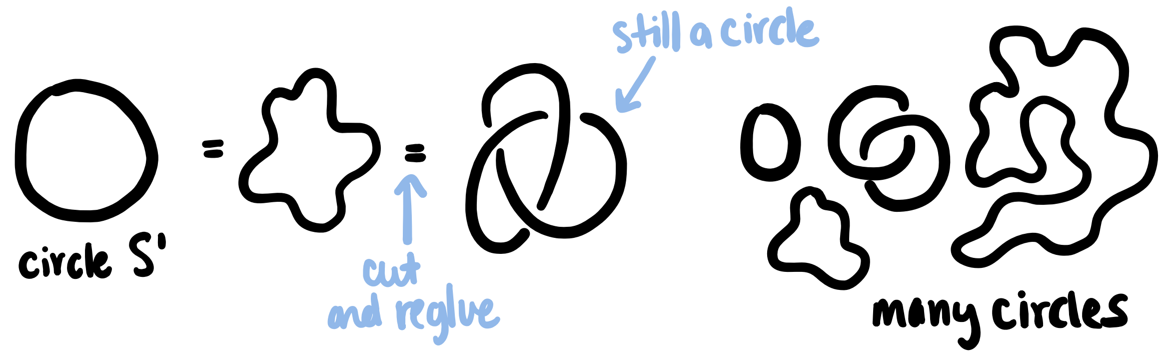

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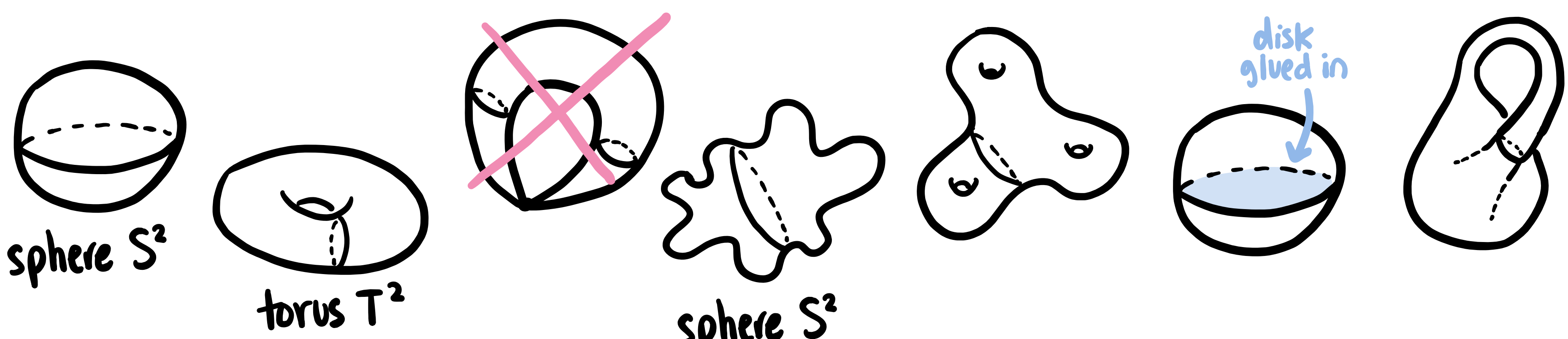

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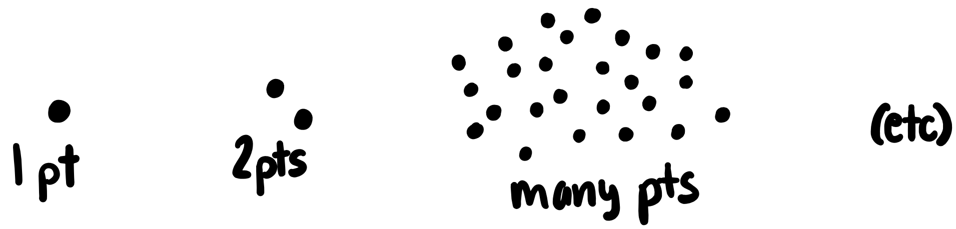

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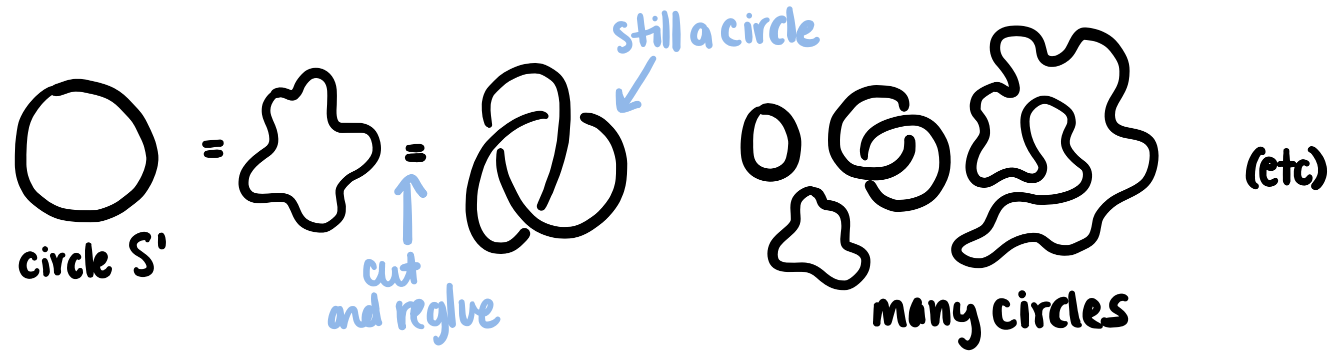

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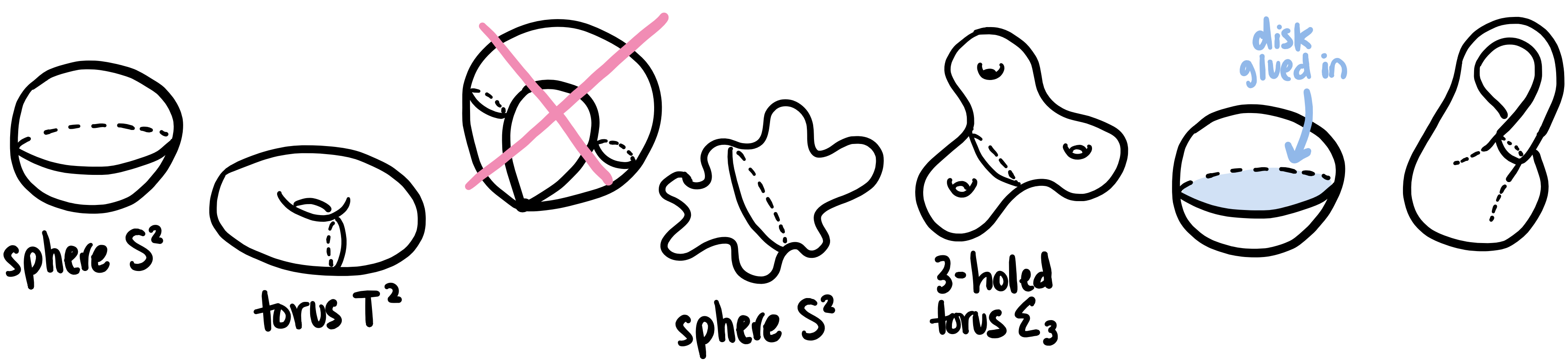

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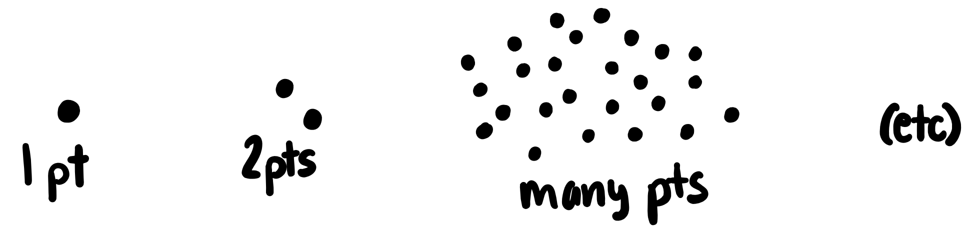

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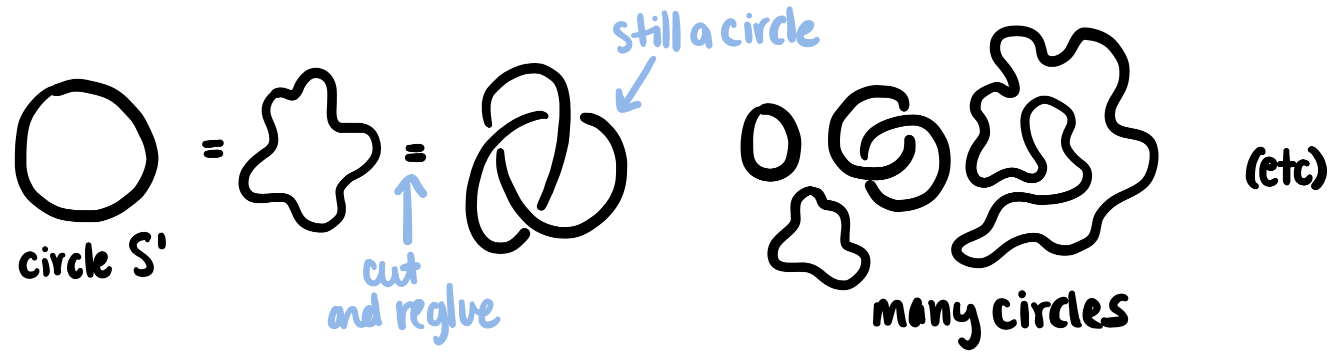

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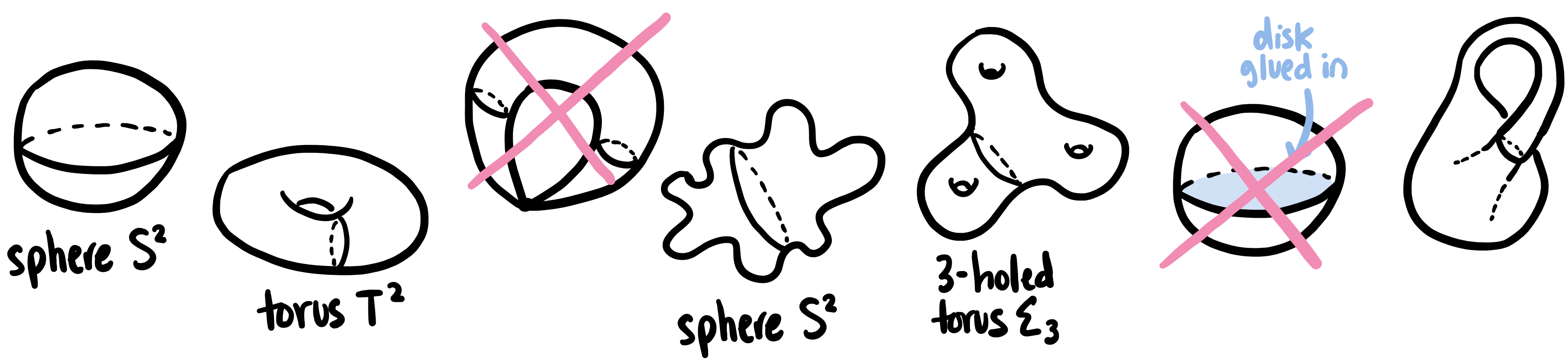

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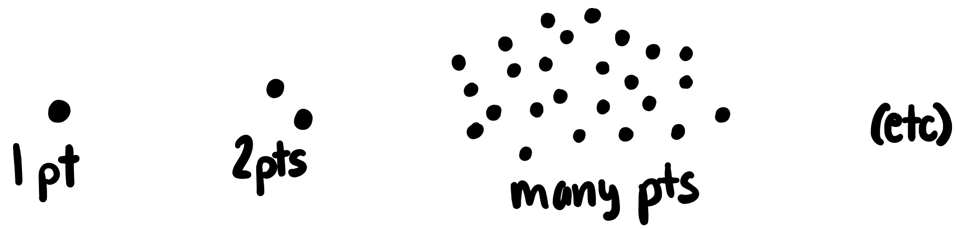

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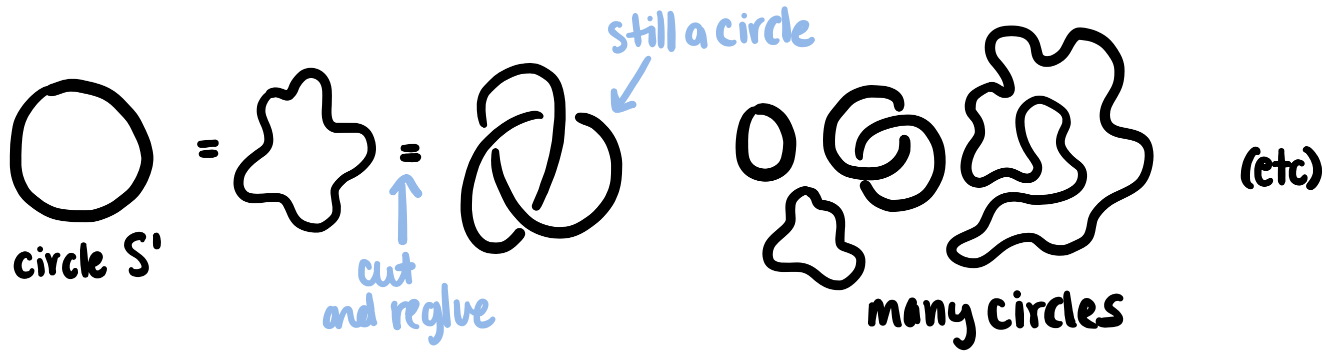

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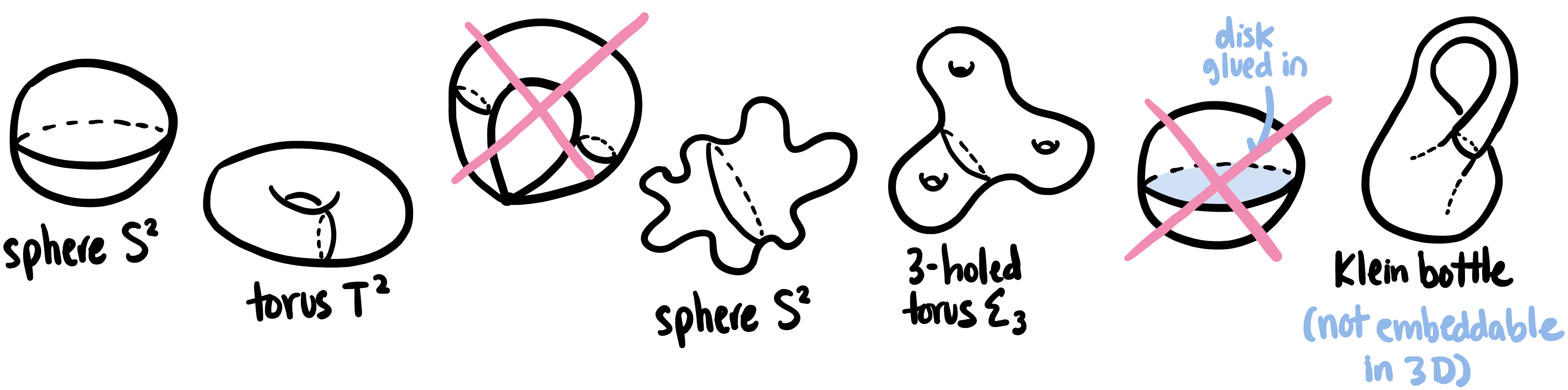

MANIFOLDS

DIMENSION 3 :

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3-sphere S^3

$\{(x,y,z,w)\in\mathbb{R}^4 \mid x^2+y^2+z^2+w^2=1\}$

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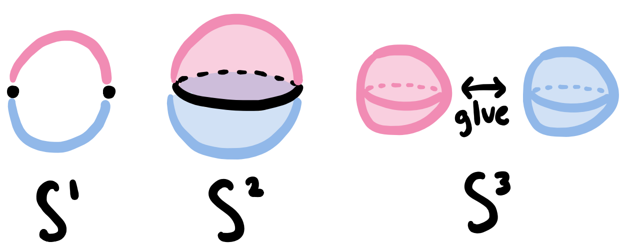

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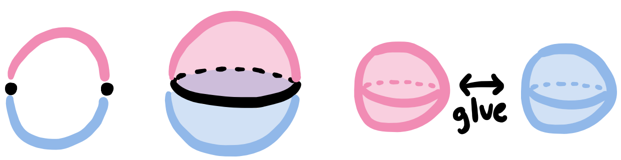

S^1 S^2 S^3

3-torus T^3

MANIFOLDS

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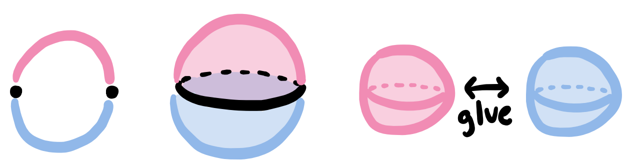

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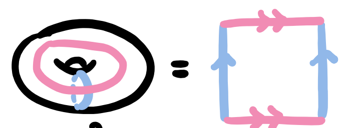

T^2

gluing diagram

MANIFOLDS

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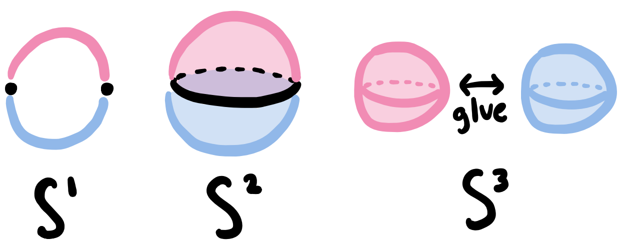

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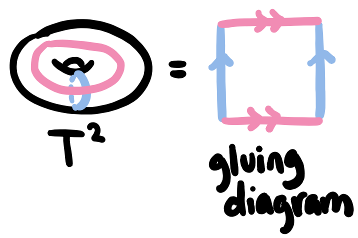

$S^1 \times S^2$

MANIFOLDS

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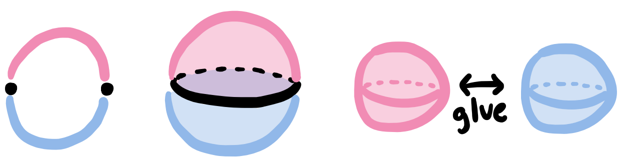

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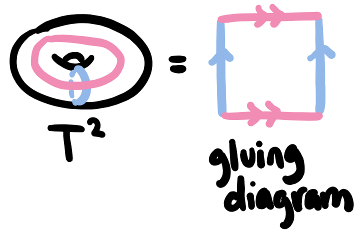

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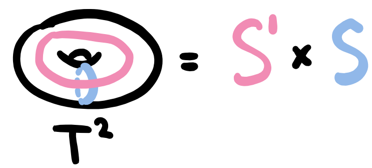

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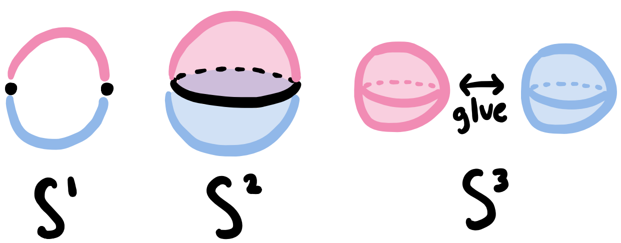

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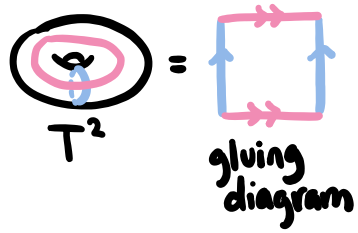

$S^1 \times S^2$

$= S^1 \times S^1$

DIMENSION 4:

MANIFOLDS

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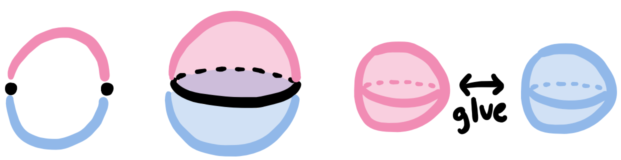

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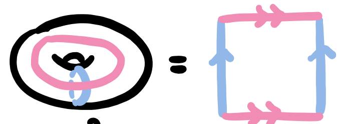
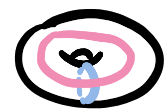


T^2 gluing diagram

$S^1 \times S^2$



$= S^1 \times S^1$

T^2

DIMENSION 4:

4-sphere S^4 4-torus T^4 $S^1 \times S^3$ $S^2 \times S^2$

MANIFOLDS

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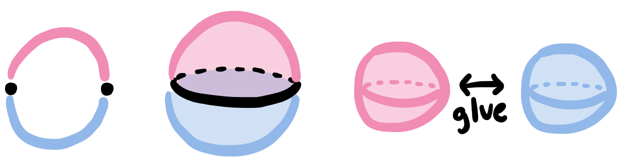

S^1 S^2 S^3

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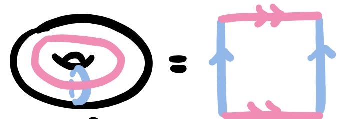

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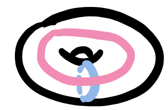
$S^1 \times S^2$



$= S^1 \times S^1$

T^2

DIMENSION 4:

4-sphere S^4 4-torus T^4 $S^1 \times S^3$ $S^2 \times S^2$

Q: how do we visualize these? can we??

Q: how do we generate other examples?

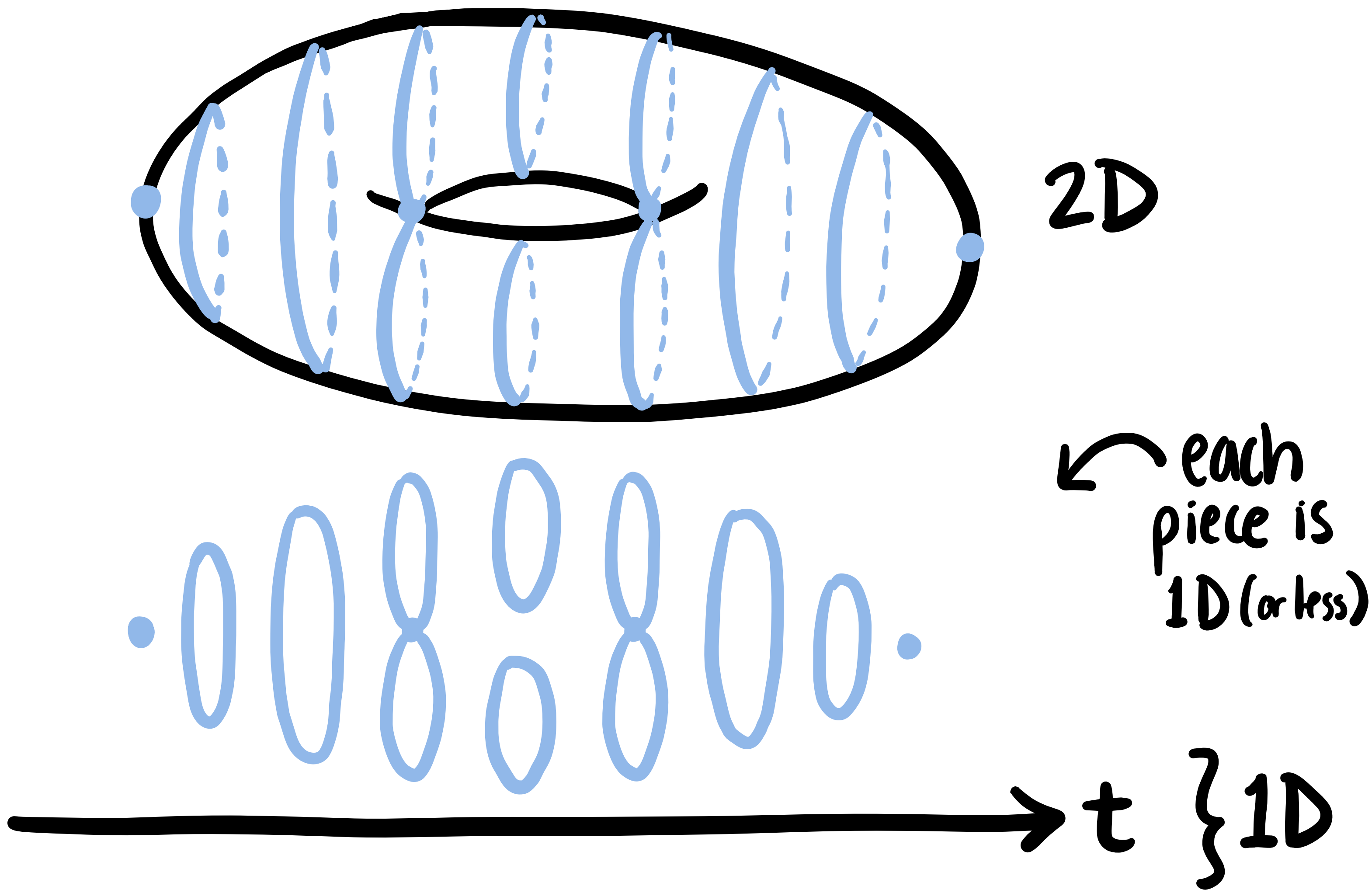
"MOVIES"

"MOVIES"

IDEA: TIME AS AN EXTRA DIMENSION

"MOVIES"

IDEA: TIME AS AN EXTRA DIMENSION



(LEFSCHETZ) FIBRATIONS

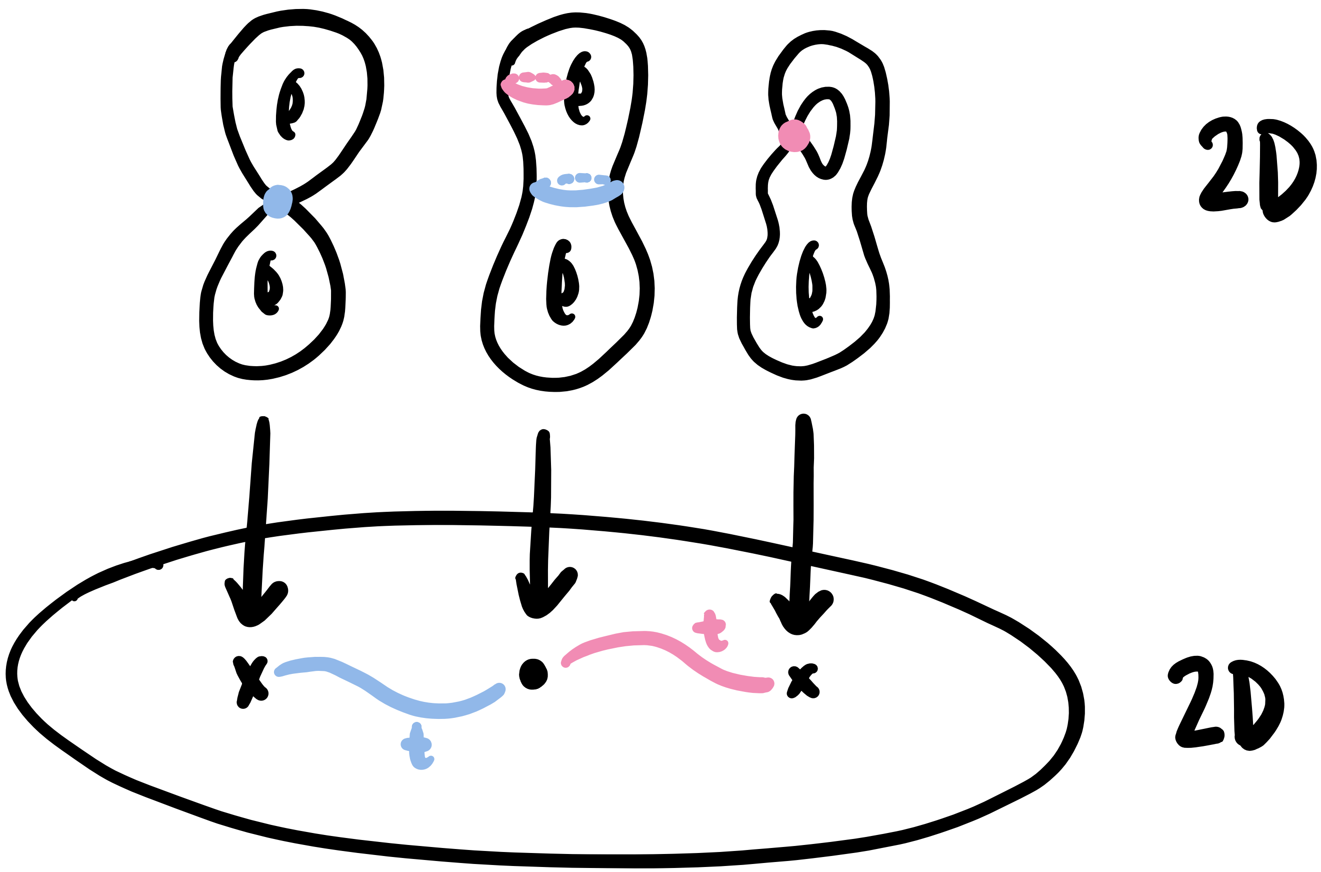
$2D \times 2D$

(rather than $3D \times 1D$)

(LEFSCHETZ) FIBRATIONS

2D × 2D

(rather than 3D × 1D)



IDEA: DECOMPOSE INTO BASIC BUILDING BLOCKS

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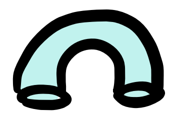
e.g. "handles" 

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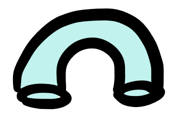
e.g. "handles" 

KIRBY DIAGRAMS

TRISECTION DIAGRAMS

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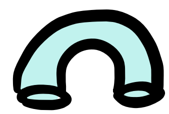
e.g. "handles" 

KIRBY DIAGRAMS

TRISECTION DIAGRAMS

TRISECTIONS

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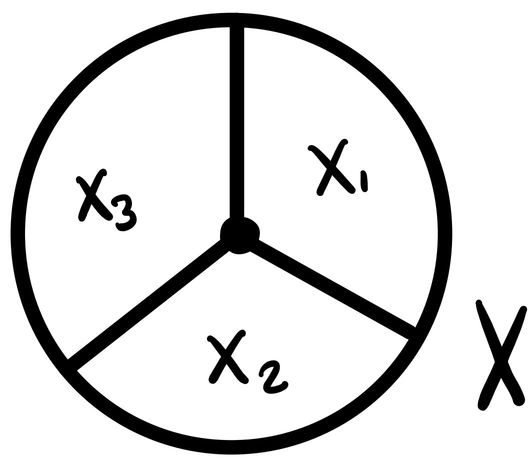
trisection of 4-manifold X [Gay + Kirby] ⤳ every closed, connected, oriented, smooth 4-manifold has a "unique" trisection



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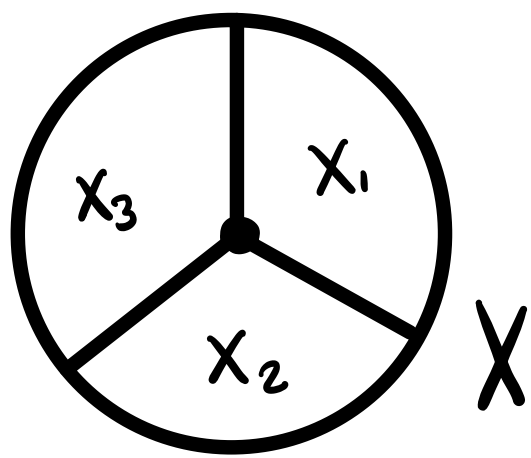

boundary connect sum

circle

3-ball

$$X_i \cong \natural^k (S^1 \times B^3)$$

"4-dim (1-)handlebody"

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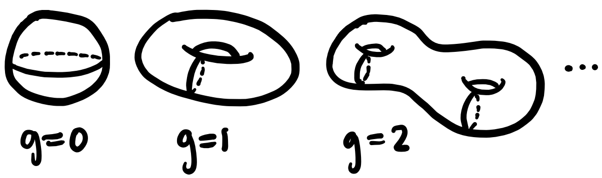
Σ_g = genus g surface



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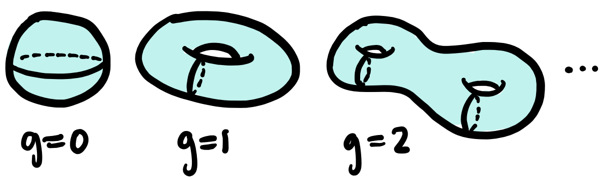

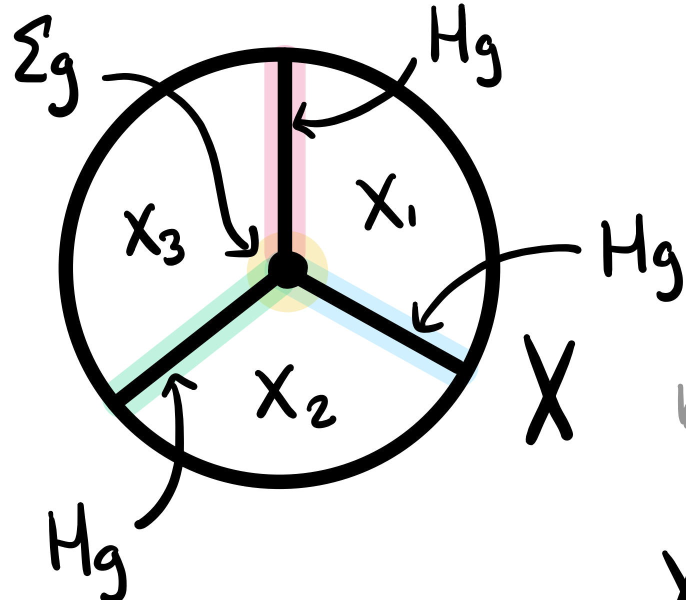

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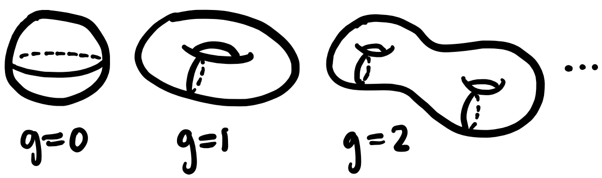

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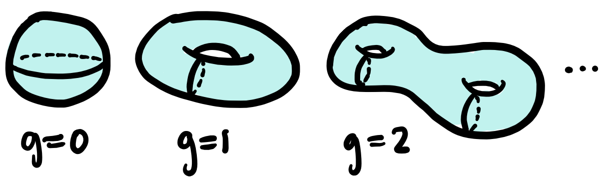

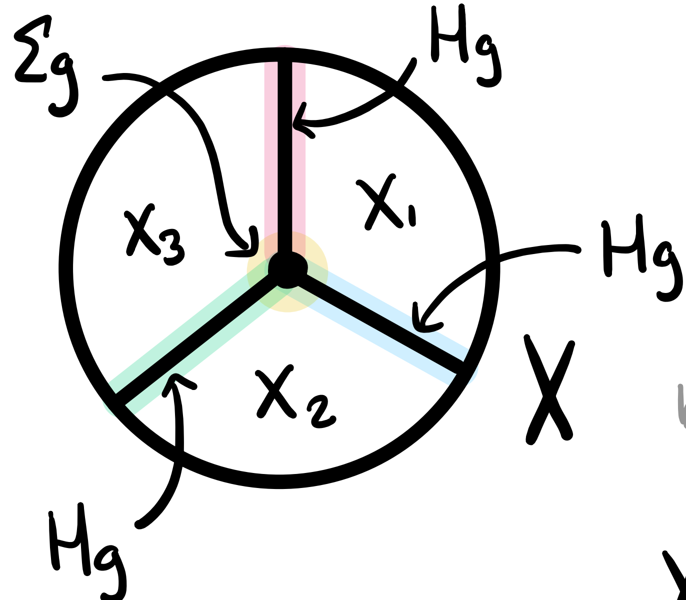

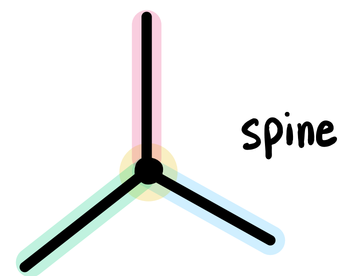

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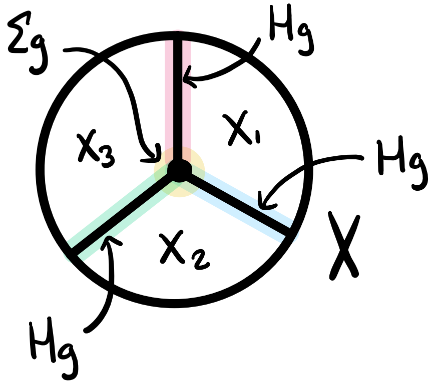
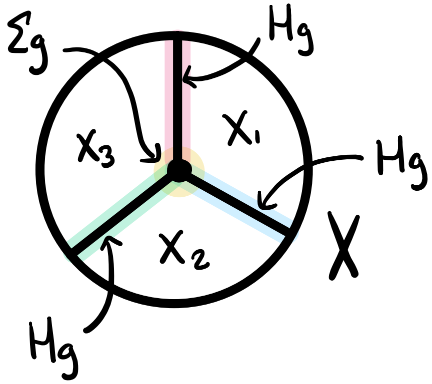
TRISECTIONS



we can represent the spine by curves on a surface

TRISECTIONS



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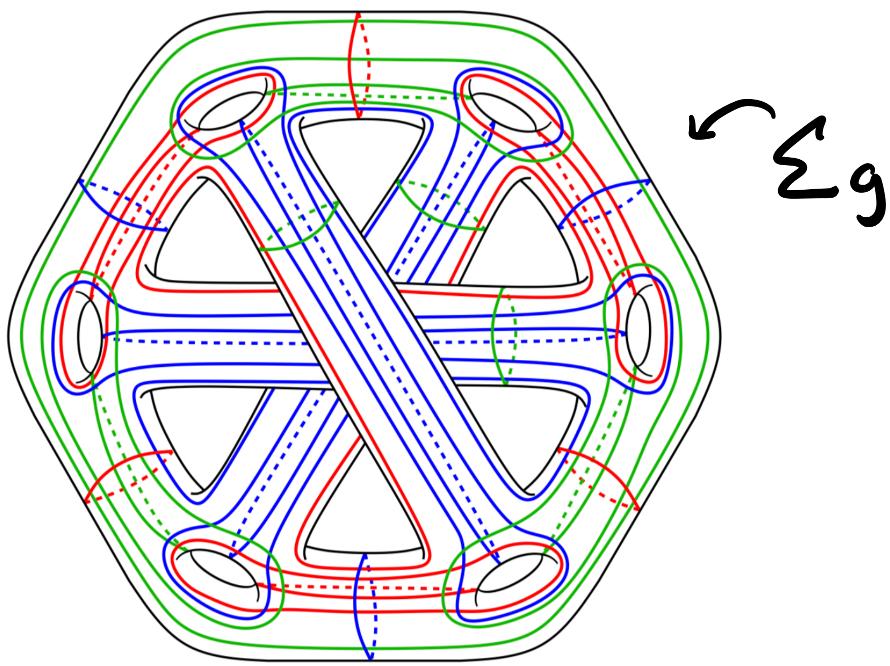
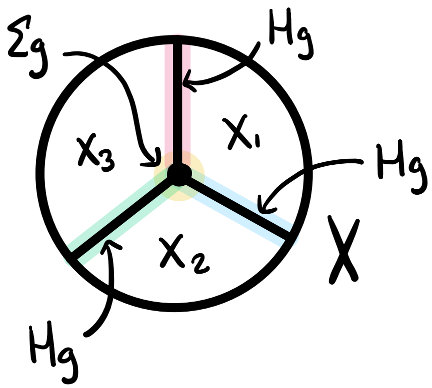


4-torus

[Marla Williams]

TRISECTIONS



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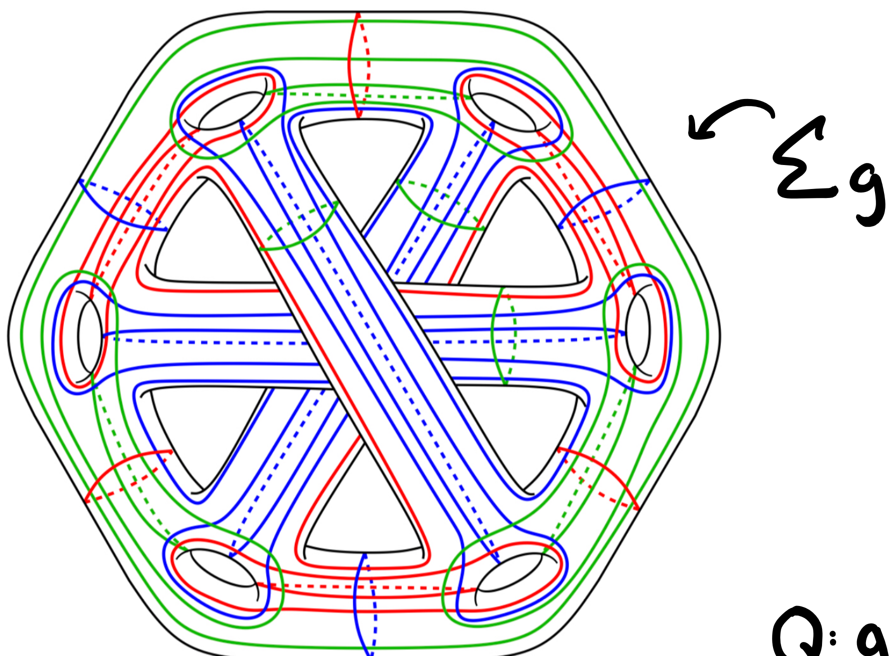

4-torus
[Marla Williams]

Q: given a 4-manifold, what is the smallest genus diagram representing it?

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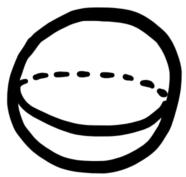
$g=0$  S^4

TRISECTIONS

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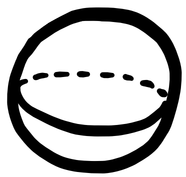
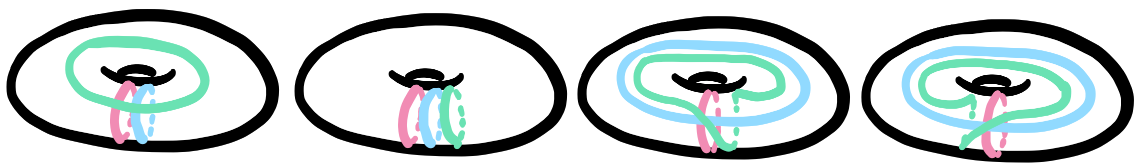


S^4

$g=1$



S^4 $\quad$ $S^3 \times S^1$ $\quad$ $\mathbb{CP}^2$ $\quad$ $\overline{\mathbb{CP}}^2$

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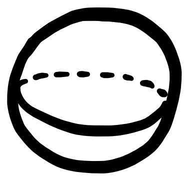

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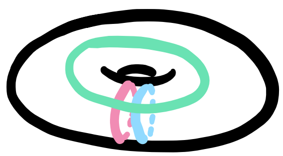
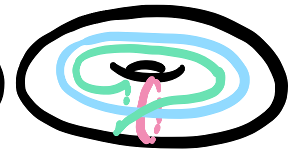


S^4

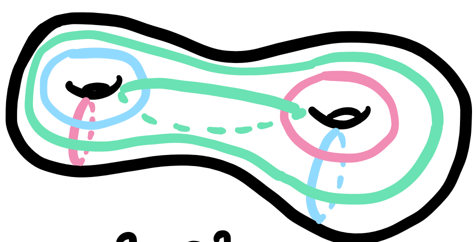
$S^3 \times S^1$

$\mathbb{CP}^2$



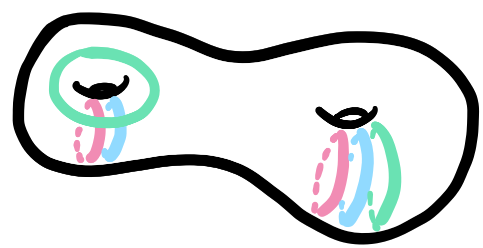
$\overline{\mathbb{CP}^2}$

$g=2$



$S^2 \times S^2$

and connect sums of ↑
[Meier+Zupan]



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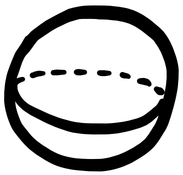
$g=0$  S^4

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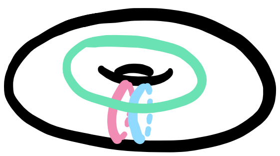

S^4

$S^3 \times S^1$

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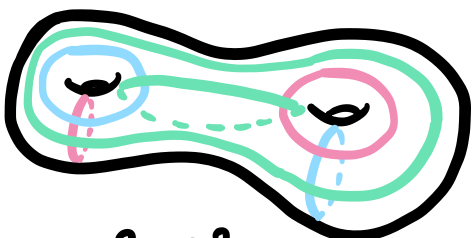

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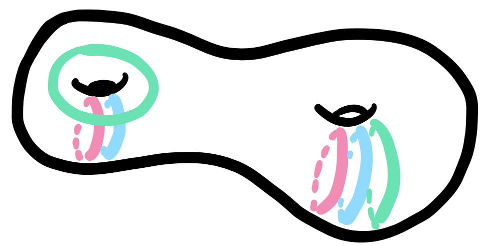

$g=3$ ∞ many! [Meier]
open... and hard!

THANKS!