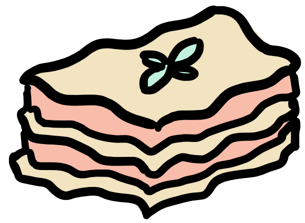

CORNERED SKEIN LASAGNA THEORY

Sarah Blackwell
University of Virginia

j.w. Slava Krushkal + Yangxiao Luo

antipasto:

OVERVIEW

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(0th degree part of "blob homology")

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"what if we could do Khovanov homology* in 4D?"

OVERVIEW

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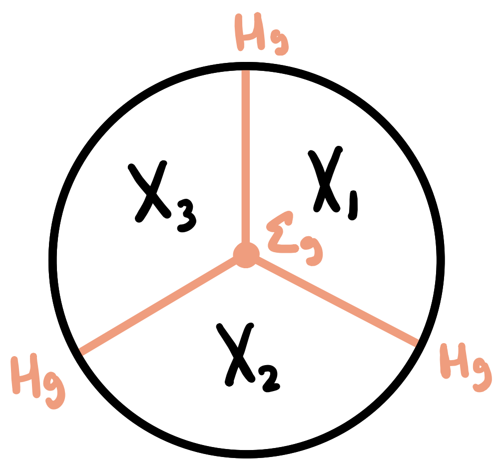

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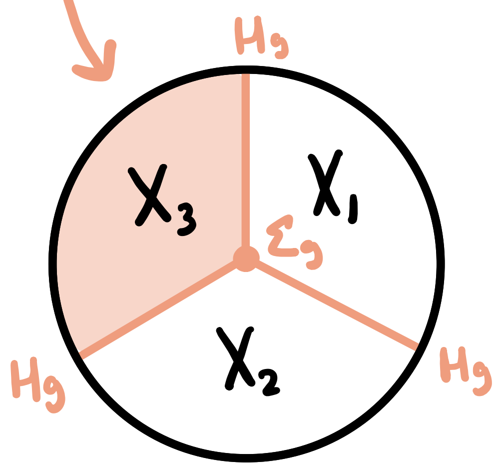

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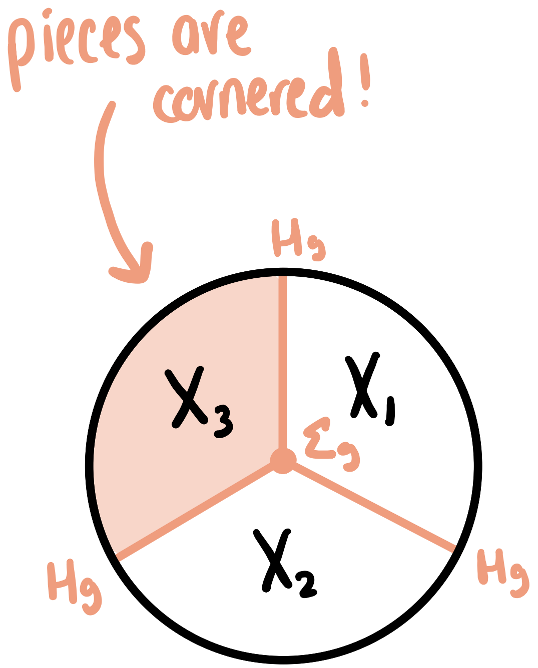

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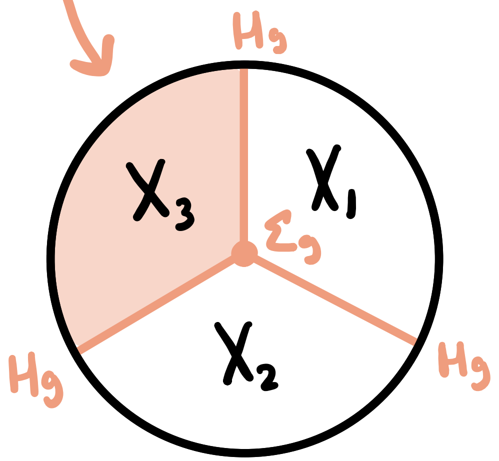

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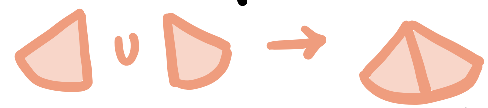
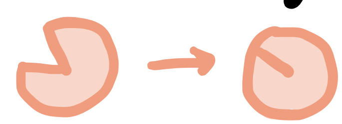
our work:

(stated/proved more generally in upcoming preprint)

- develop categorical framework
- define cornered skein lasagna invariant
- gluing theorem 1: two pieces along common boundary



- gluing theorems 2+3: self-gluing along boundary



primo:

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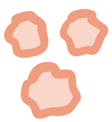

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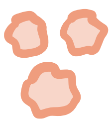

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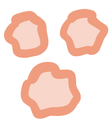

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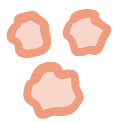

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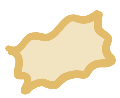

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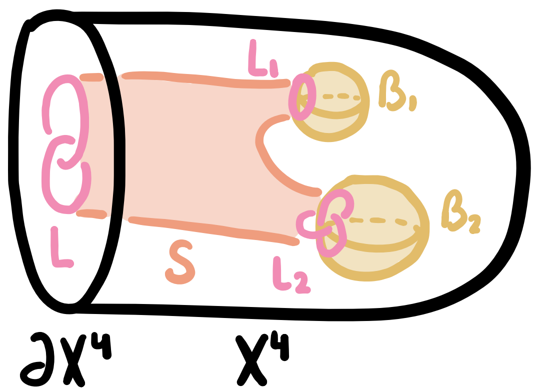

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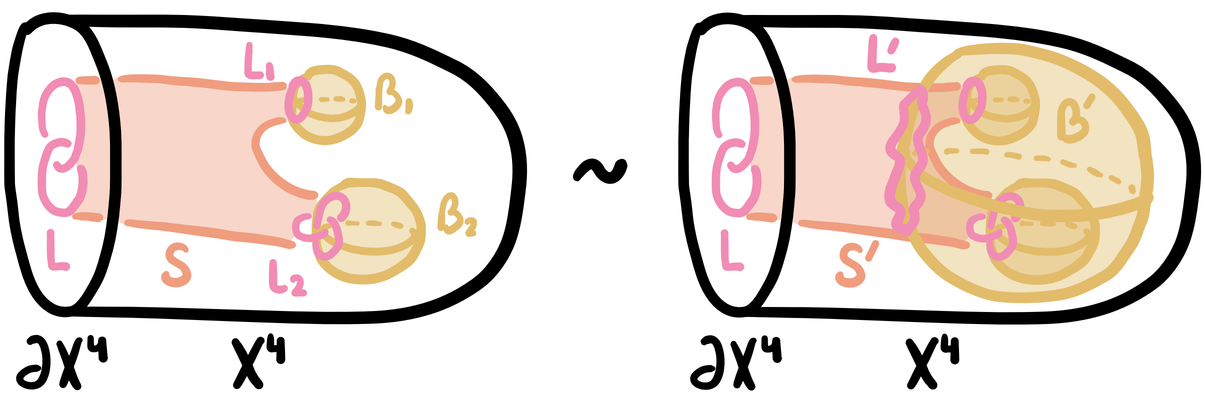

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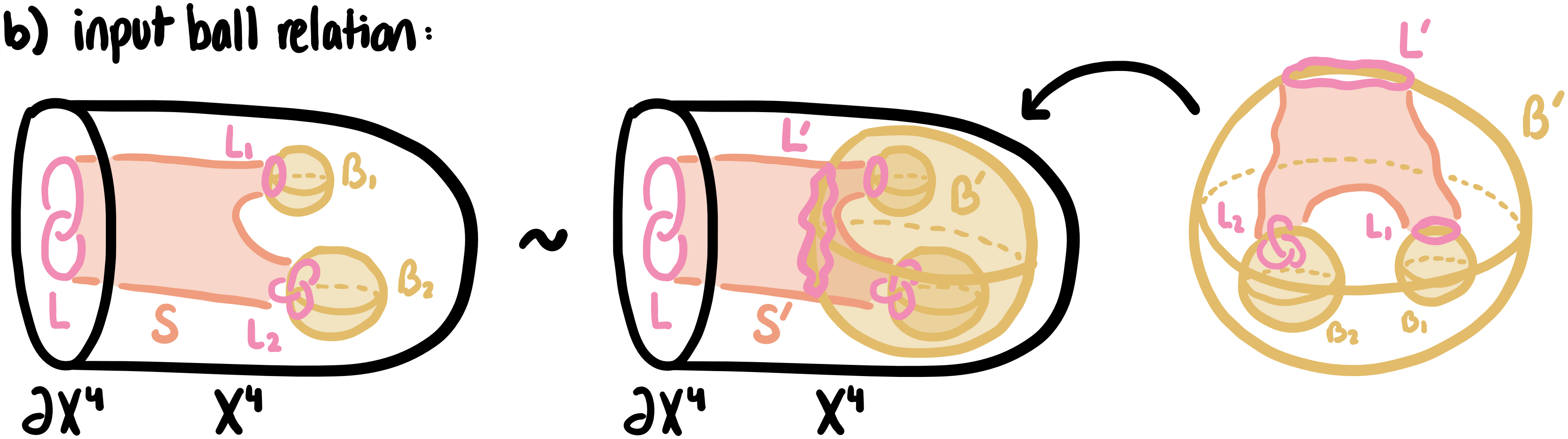

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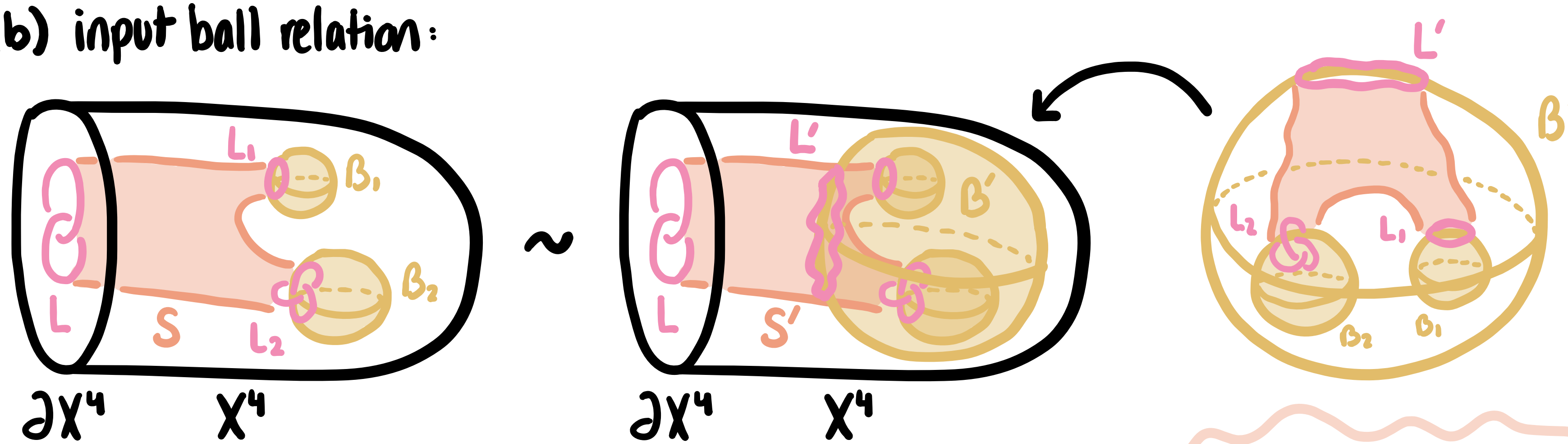

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when $X=B^4$, $\mathcal{S}(X,L)$ recovers $KhR(L)$

secondo:

OUR WORK

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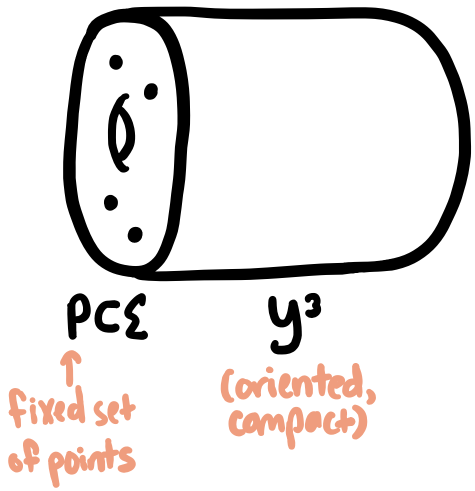

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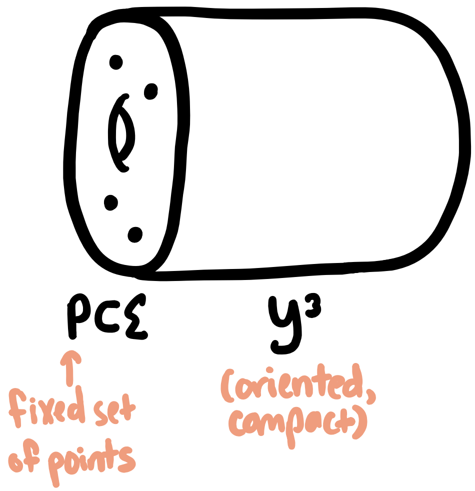

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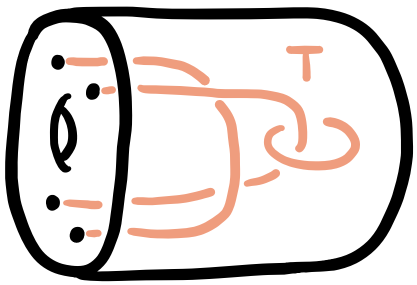

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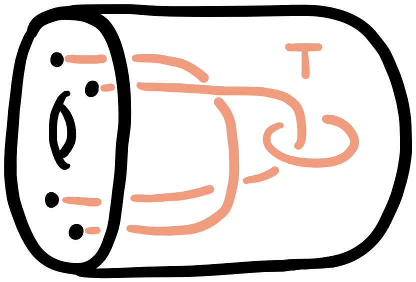

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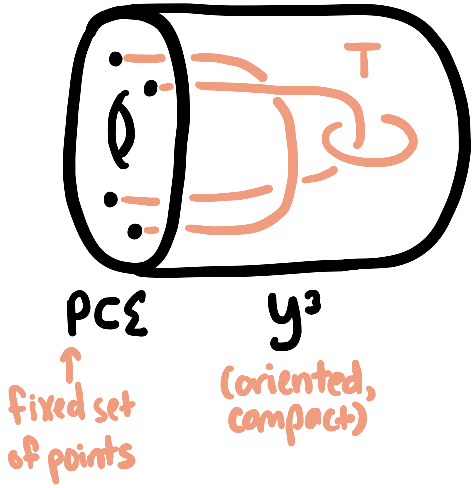

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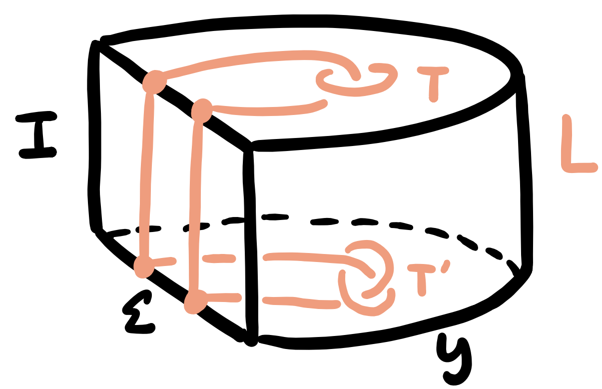

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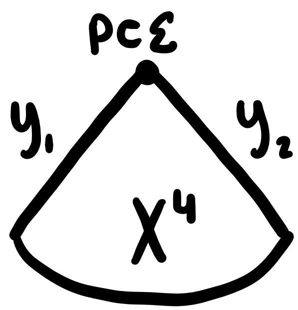

cornered 4-manifold

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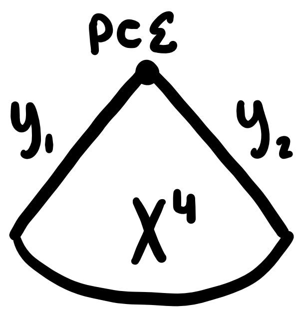

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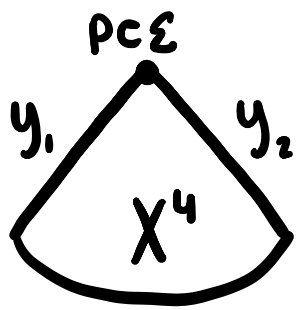

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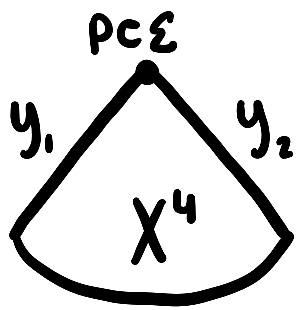

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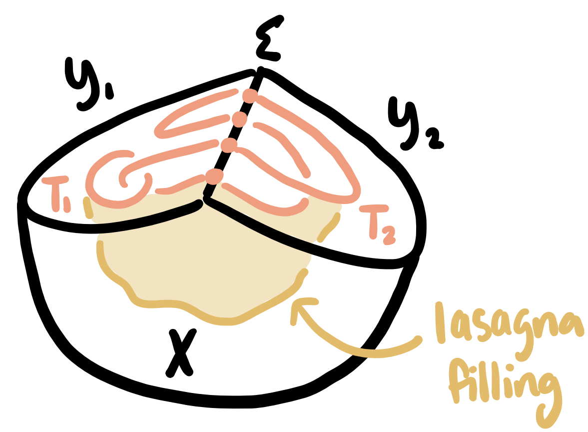

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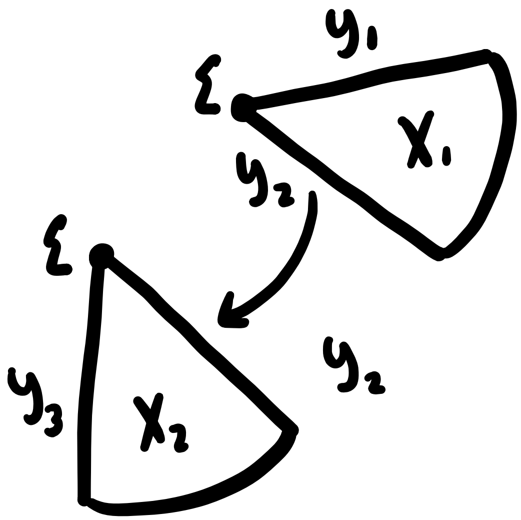

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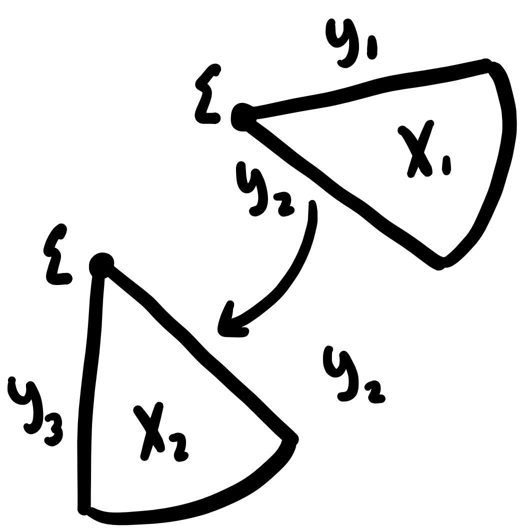

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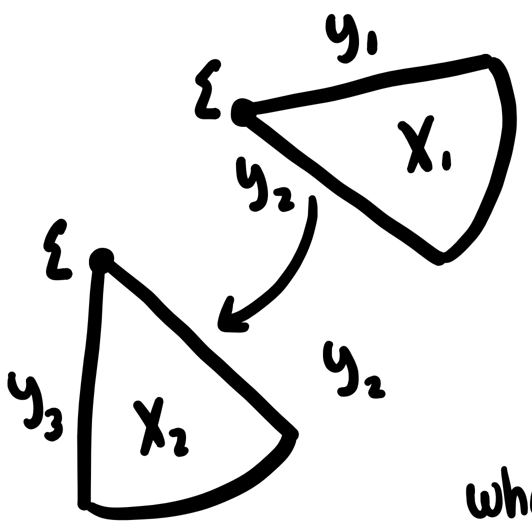

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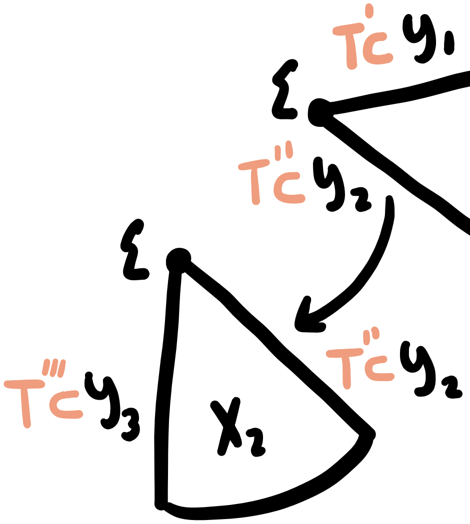

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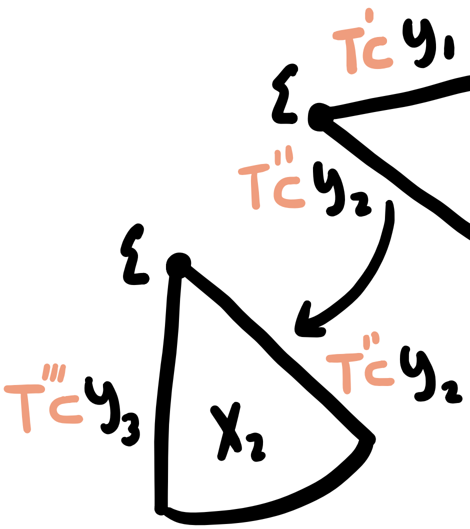

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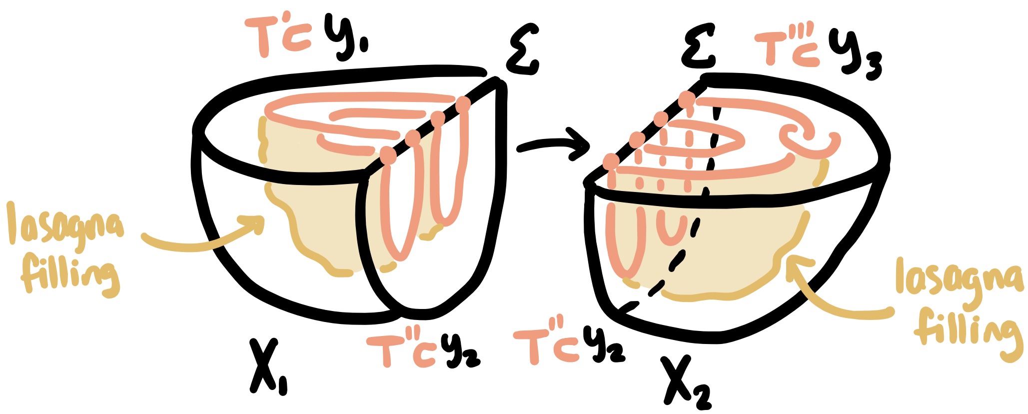
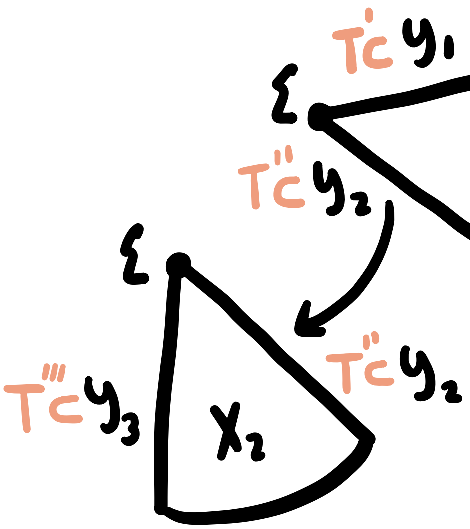


OUR WORK



gluing two pieces

given $F_{X_1,P}$ and $F_{X_2,P}$, we define their tensor product:

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$$(a\cdot\beta)\otimes c \sim a \otimes (\beta\cdot c)$$

lasagnas! $\begin{cases} a \in S(X_1, T'\cup \tilde{T}'') \\ c \in S(X_2, T''\cup T''') \\ \beta \in \mathrm{Mor}(\tilde{T}'', T'') \end{cases}$

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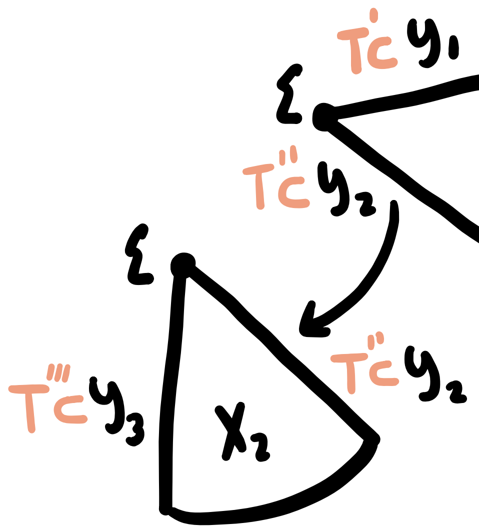

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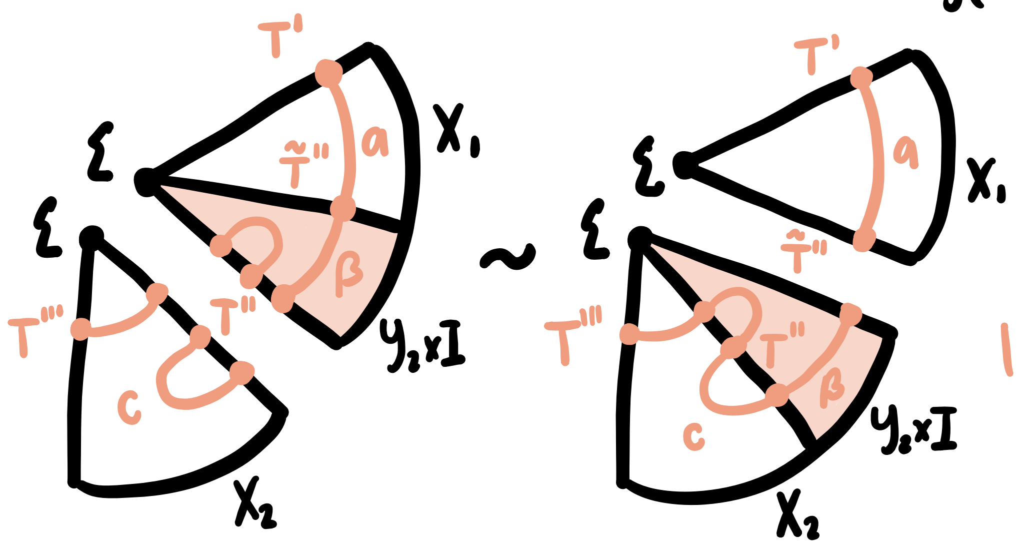

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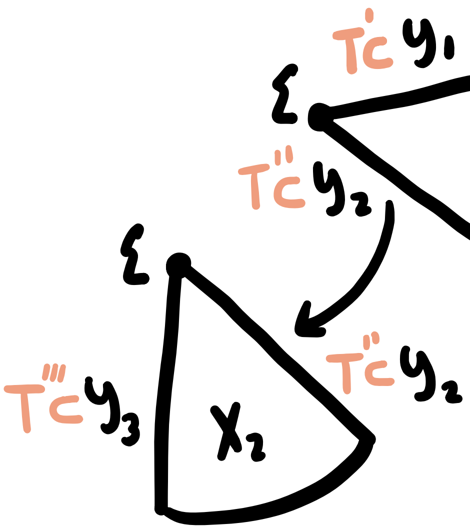

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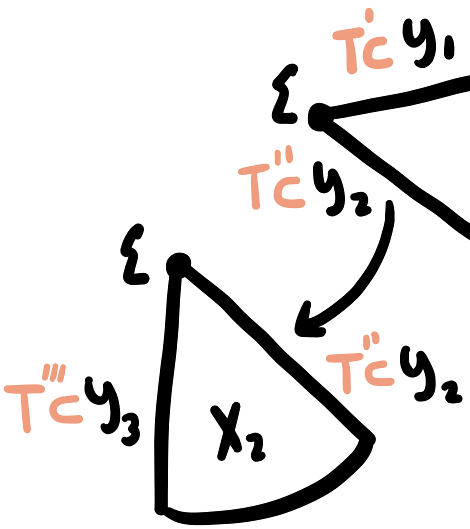
Theorem [BKL]: the map $F_{X_1,P} \underset{S(y_2,P)}{\otimes} F_{X_2,P} \longrightarrow \overbrace{F_{X_1 \cup_{y_2} X_2}}^{S(X_1 \cup X_2,-)}$ is an isomorphism

$$a \otimes c \longmapsto a \cup c$$

OUR WORK



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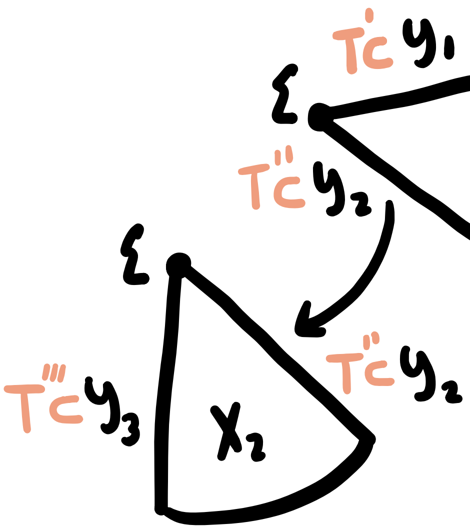
OUR WORK



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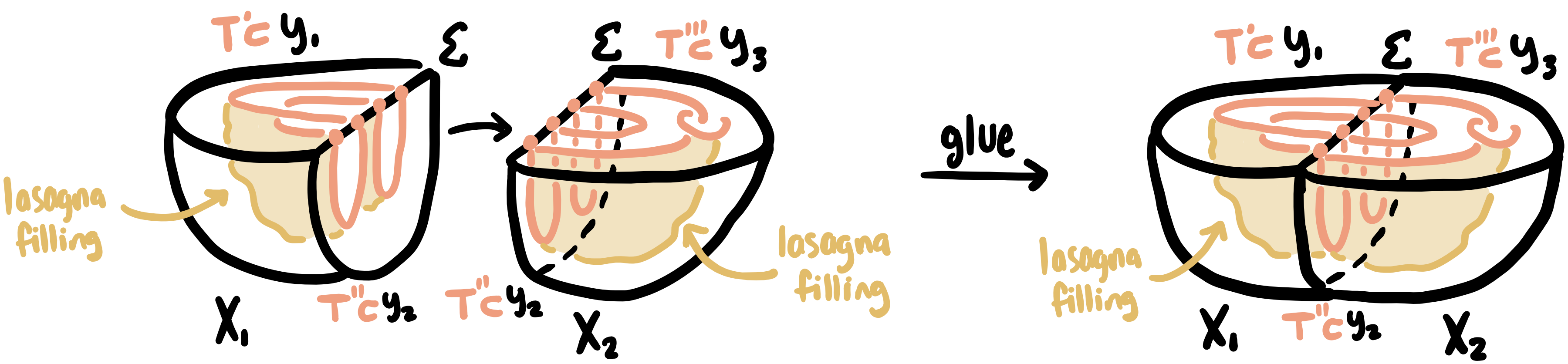

OUR WORK

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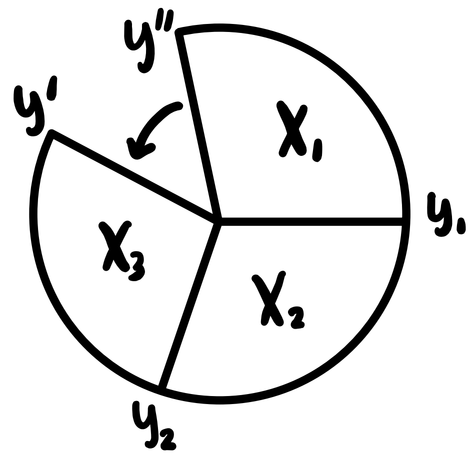

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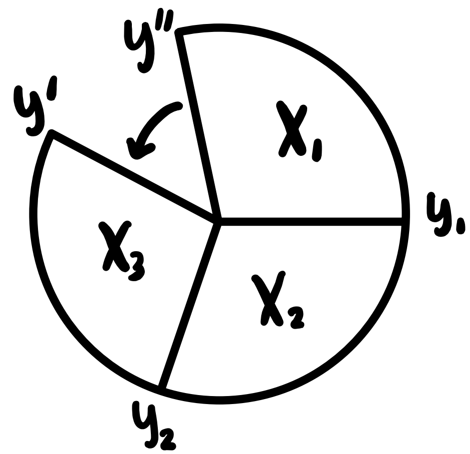

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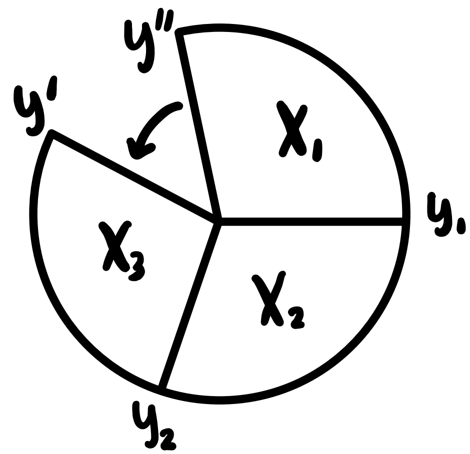

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aka getting a gluing theorem

modulo some (many?) technicalities, this process is very similar to that for the preceding theorem

OUR WORK

self gluing

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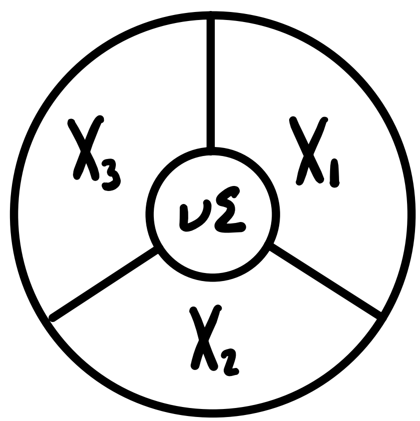
self gluing

Theorem [BKL]: $\underbrace{\langle F_{x,p} \rangle}_{\text{a self-tensor product}} \cong S(\underbrace{\bar{X} \setminus \nu(\Sigma)}_{\text{for technical reasons it's advantageous to break into two parts: (1) remove } \nu(\Sigma)\text{, (2) put back in}}, P \times S')$



OUR WORK

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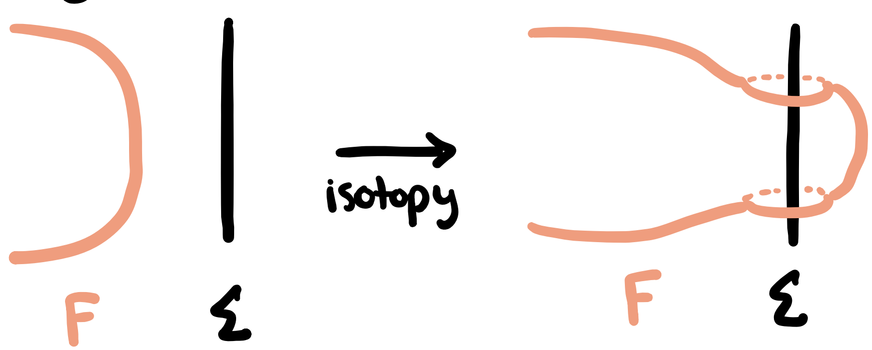
(b) the following relation "across" Σ:



dolce :

CONCRETE CALCULATIONS

(TBD)

THANKS!