TRISECTIONS FACT SHEET

* all 4-mflds are smooth, connected, and compact *

Trisectors Workshop 2023

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OVERVIEW / CONTEXT

"from Heegaard splittings to trisections" [Gay '18]

TRISECTIONS OF...

4-manifolds

- closed, orientable =: ⊛ [Gay + Kirby '16]
- orientable w/ boundary [Castro '16 + '17] [Castro, Gay, Pinzón-Caicedo '18] (x2)
- closed, non-orientable [Miller + Naylor '20]

knotted surfaces

- in S^4 [Meier + Zupan '17]
- in ⊛ 4-mflds [Meier + Zupan '18] [Hughes, Kim, Miller '20]
- in ⊛ 4-mflds w/ boundary [Meier '20]
- in rational surfaces [Meier + Lambert-Cole '22]

other dimensions

- 3-mflds [Koenig '18]
- 5-mflds [Miller + Lambert-Cole '21]
- PL n-mflds [Rubinstein + Tillmann '18]

CONCRETE EXAMPLES (⊛)

- S^4, $\mathbb{CP}^2$, $\overline{\mathbb{CP}}^2$, $S^1 \times S^3$, $S^2 \times S^2$, $S^2 \tilde{\times} S^2$ [Gay + Kirby '16]
- spun lens spaces [Meier '18]
- 3-mfld bundles over S^1 [Koenig '21]
- flat surface bundles over surfaces [Williams '20]

CLASSIFICATION MATTERS (⊛)

- via (minimal) trisection genus:
  - $g=0$ S^4
  - $g=1$ $\mathbb{CP}^2$, $\overline{\mathbb{CP}}^2$, $S^1 \times S^3$
  - $g=2$ $S^2 \times S^2$ (+ connect sums of above) [Meier + Zupan '17]
  - $g=3$ open [Meier '18] [Aranda + Moeller '20]
- ∃ distinct trisections of the same genus on a fixed 4-mfld ("stabilization is required") [Islambouli '21]

MULTISECTIONS

(generally: more pieces ↔ lower genus)

- ⊛ 4-mflds: ≥ 3 pieces
  ⊛ 4-mflds w/ boundary: 2 pieces [Naylor + Islambouli '20]
- curves in rational surfaces [Islambouli, Karimi, Lambert-Cole, Meier '22]
- higher dimensions (PL) [Aribi, Courte, Golla, Moussard '23]

GROUP TRISECTIONS OF...

group theoretic version of trisections →

π_1 (⊛ 4-mflds)

[Abrams, Gay, Kirby '18]

π_1 (knotted surfaces in ⊛ 4-mflds)

[Blackwell, Kirby, Klug, Longo, Ruppik '23]

INVARIANTS FROM TRISECTIONS

— classical invariants —

- homology + intersection form (trisections of 4-mflds) [Feller, Klug, Schirmer, Zemke '18]
- twisted homology, Reidemeister torsion, intersection form (trisections of 4-mflds) [Florens + Moussard '22]
- twisted homology, intersection forms, etc. (multisections of 4-mflds) [Moussard + Schirmer '21]
- homology (trisections of 4-mflds w/ boundary) [Tanimoto '23]
- Ozsváth-Szabó cobordism invariants (trisections of 4-mflds w/ boundary) [Olsen '21]
- classical knotted surface invariants (bridge trisections of knotted surfaces) [Joseph, Meier, Miller, Zupan '22]

— Kirby-Thompson $\mathcal{L}$-invariants —

roughly:
an invt. measuring complexity of a mfld as determined by the complexity of trisection diagrams representing it (generally hard to calculate ☹)

- for ⊛ 4-mflds [Kirby + Thompson '18]
- for ⊛ 4-mflds w/ boundary [Castro, Islambouli, Miller, Tomova '22]
- for knotted surfaces in S^4 [Blair, Campisi, Taylor, Tomova '22]

SYMPLECTIC CONNECTIONS

- Weinstein trisections [Lambert-Cole, Meier, Starkston '21]
  - the symplectic form induces a Weinstein structure on each piece of the trisection
  - bridge position for symplectic surfaces in $\mathbb{CP}^2$

- trisections of Lefschetz pencils [Gay '16]
- trisections of 4-mflds via Lefschetz fibrations [Castro + Ozbagci '19]
- simplified broken Lefschetz fibrations ↔ simplified trisections of 4-mflds [Baykur + Saeki '18]
- characterization of symplectic surfaces in $\mathbb{CP}^2$ via bridge trisections [Lambert-Cole '19]
- hexagonal lattice diagrams for complex curves in $\mathbb{CP}^2$ [Zupan '22]
- criteria on a trisection to construct a symplectic structure [Lambert-Cole '22]
- Weinstein 4-mfld ⇝ multisection diagram w/ divides [Islambouli + Starkston '23]
- encoding Lagrangians in $\mathbb{CP}^2$ by trisections [Blackwell, Gay, Lambert-Cole (very soon...)]